Quark spectroscopy and Cabibbo angle

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(Submitted 10 April 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 10, 599–602 (20 May 1978)

On the basis of a modification of an idea by Weinberg and Fritzsch, a formula is derived for the Cabibbo angle θ_c in terms of the strong-interaction symmetry-breaking parameter r. The value $\theta_c \approx 12.8°$ obtained from the stationarity condition $d\theta_C/dr = 0$ agrees well with the experimental value of θ_C and with the empirical masses of the structure quarks.

PACS numbers: 12.30.–s, 11.40.–q

Ever since the concept of rotation of the hadron current through the Cabibbo angle has been introduced in physics and restored the universality of the weak interaction, there have been unceasing attempts to calculate this angle or at least to explain its smallness ($\theta_C \approx 13°$). The idea of a theoretical estimate of θ_C is suggested by the empirical relation $\tan\theta_c \approx m_\pi/m_k$, from which it follows apparently that the value of θ_C is connected with the breaking of the strong-interaction symmetries. It was shown in[1,2] how the requirement of cancelling out the divergences in the mass matrix of quarks induced by weak interaction can lead to a relation of the type $\tan^2\theta_C \sim d/s$, where the symbols d and s stand for the masses of the corresponding quarks. In this approach, the Cabibbo angle is essentially connected with the $SU_{2L} \times SU_{2R}$ symmetry of strong interaction, and the smallness θ_C is attributed to smallness of the breaking of this symmetry.

The invention of the c-quark has made it possible to make the weak interaction more symmetrical[3] and has uncovered new possibilities for the calculation of θ_C. The quark pairs (uc) and (ds) can be subjected to independent rotations through angles θ_1 and θ_2, respectively, and the hadron current $J = \bar{c}_{\theta_1} s_{\theta_2} = \bar{u}_{\theta_1} d_{\theta_2}$ can be represented in standard form $J = \bar{c} s_{\theta_C} + \bar{u} d_{\theta_C}$, where $\theta_c = \theta_2 - \theta_1$, and $d_{\theta_C} = d \cos\theta_C + s \sin\theta_C$, etc. By using these rotations and certain assumptions concerning the form of the mass matrix Q_{ij} of the quarks in one of the bases, Weinberg[4] and Fritzsch[5] arrived at formulas that express θ_C in terms of u, d, s, and c and reduce at $u/c \ll d/s \ll 1$ of the corresponding pairs (uc) and (ds) satisfy the conditions

$$Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0, \quad Q^{(1)}_{uc} = \overset{*}{Q}{}^{(1)}_{cu}, \quad Q^{(2)}_{ds} = \overset{*}{Q}{}^{(2)}_{sd} .$$

It follows therefore that their determinants are negative ($\det Q^{(1)} = -|Q^{(1)}_{uc}|^2$), i.e., the eigenvalues, which should coincide with the masses of the quarks, have different signs (say, $d < 0$). These masses are identified with the ($\pm$) masses of the "current" quarks.

We shall show first that by using other natural hypotheses, which are closer to the ideas of[1,2], it is possible to obtain for θ_C expressions analogous to the Weinberg–Fritzsch formulas. Obviously, when the weak interaction is turned off the matrices $Q^{(1)}$ and $Q^{(2)}$ are diagonal, and their eigenvalues coincide with the masses of the quarks (uc), (ds) which determine the masses of the observed mesons (the masses of the "structure" quarks). Turning on a weak interaction makes these matrices nondiagonal:

$$Q^{(1)}_{uc} = Q^{(1)}_{cu} = a_1, \; Q^{(2)}_{ds} = Q^{(2)}_{sd} = a_2,$$

and $a_1 a_2$ can be chosen real (cf.[5]). We assume now that the weak interaction "selects" a basis $(u_{\theta_1} c_{\theta_1})(d_{\theta_2} s_{\theta_2})$, such that the matrices $Q^{(i)}$ are diagonal and $Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0$. This requirement means, in essence, that the masses of the "current" quarks u and d are equal to zero, and that exact $SU_{2L} \times SU_{2R}$ symmetry is obtained in the new basis. Elementary diagonalization with allowance for the conditions $Q^{(1)}_{uu} = Q^{(2)}_{dd} = 0$ leads to the results

$$\tan 2\theta_1 = 2\sqrt{cu}\,(c - u)^{-1}, \tan 2\theta_2 = 2\sqrt{sd}\,(s - d)^{-1}. \tag{1}$$

Recognizing that $\theta_C = \theta_2 - \theta_1$, we obtain the Fritzsch formula, which we represent in the form

$$\tan\theta_C = (r - r_1)(1 + r r_1)^{-1}, \quad r^2 \equiv d/s, \; r_1^2 \equiv u/c. \tag{2}$$

If we add a_i also to the diagonal elements of the corresponding matrices $Q^{(i)}$, we arrive as a result to the same formula (2) for θ_C, but with $r \to \bar{r} = d/s, r_1 \to \bar{r}_1 = u/c$. If the corrections due to the weak interactions have a different structure, then formula (2) may remain in force, but the connection of r and r_1 with ratio of the quark masses will be more complicated [for example, by adding a_i to all the elemtns of $q^{(i)}$ except $Q^{(1)}_{uu}, Q^{(2)}_{dd}$, we easily obtain the indicated generalized relations (2)]. The form of the "weak" corrections is determined by the choice of the concrete model of the weak interaction. As indicated in[4,5] and in the papers cited therein, it is quite probable that in renormalizable theories based on the group $SU^W_{2L} \times SU^W_{2R} \times U^W_1$ this matrix is of a form close to that required by us.

The hypothesis that the "current" masses u and d are equal to zero has led to the nontrivial relations $a_1 = \sqrt{cu}$, $a_2 = \sqrt{sd}$, which in principle can be satisfied in unified theories of strong, electromagnetic, and weak interactions.[4] Since there is still no real possibility of calculating the quark masses and the corrections a_i in such theories, we attempt to use the necessary condition for the existence of self-consistent solution of our equations—the principle of stationarity of the Cabibbo angle with respect to small variations of the parameter that describes the breaking of the strong-interaction symmetry. We can choose this parameter to be r, and then $r_1 = r_1(r)$. This assumption can be approximately realized in the theory of spontaneous breaking of $U_{nL} \times U_{nR}$, in which there exist, besides the "diagonal" quark transitions $\mathfrak{M}(q_{i\bar{j}} \to q_i \bar{q}_j) \sim g_D$, also "nondiagonal" $\mathfrak{M}(q_{i\bar{i}} \to q_j \bar{q}_j) \sim g_E$, with $g_E/g_D \ll 1$ (for details see[6]). The asymmetrical solutions of the self-consistent-field equations for the quark propagators turn out under certain conditions to be more stable than the symmetrical ones and satisfy the approximate relation

$$\frac{u}{s} \sim \frac{d}{s} \sim \frac{s}{c} \sim \frac{g_E}{g_D}.$$

The same mechanism of mixing the quarks explains simultaneously the strong $\eta-\eta'$ mixing ($\sim\epsilon_\eta^2$), with $(\epsilon_\eta^2/m_k^2)\sim(g_E/g_D)$ [we obtained from the empirical formulas $(\epsilon_\eta^2/m_k^2)\sim(1/5)$].[7]

The requirement that θ_C be stationary follows from the fact that the parameters a_1 are themselves implicitly dependent on θ_C (for example, in the simplest model[2] $a_1\sim\sqrt{uc}\ \sin 2\theta_C$). Therefore a self-consistent solution is possible only if θ_C depends weakly on r. The stationarity condition can be understood also more formally. If no free quarks exist, then their masses and the parameter r are determined by a certain averaging process[8] with a limited accuracy ($\sim\delta m,\delta r$). The need for a weak dependence of θ_C of small variations of r, which is formally expressed by the equality $d\theta_C/dr=0$, is obvious.

The condition $\theta'(r_0)=0$ now makes it possible to determine r_0 and $\theta_C(r_0)$:

$$r_1'(r_0) = [1+r_1^2(r_0)](1+r_0^2)^{-1},\quad \mathrm{tg}\,\theta_C(r_0) = [r_0 - r_1(r_0)](1 + r_0 r_1(r_0))^{-1}. \tag{3}$$

By making the simplest assumption $r_1=r^2$, we can obtain from (3) the brilliant formula

$$r_0 = \frac{1+\sqrt{3}}{2}\left(\frac{3}{4}\right)^{1/4} = 4354,\ \tan^2\theta_C(r_0) = \frac{\sqrt{1/3}-1/3}{\sqrt{3}+3},\ \theta_C(r_0) = 12{,}794^\circ. \tag{4}$$

The assumption that led us to (4) agrees well with the empirical relation between u/c and d/s obtained in[7]. It is possible to reproduce roughly the result of that reference by putting $q_{i\alpha}^2=q_i^2+m_\alpha^2$, where $q_{i\alpha}$ is the effective mass of the quark in the vector ($\alpha=V$) or pseudoscalar ($\alpha=P$) meson, and q_i is the true mass of the i-th structure quark. Then, if we neglect the mixing of the quarks, the mass of the meson $(q_i\bar{q}_j)_\alpha$ is equal to $M_{ij\alpha}=q_{i\alpha}+q_{j\alpha}$ (see[6,7]) and, using the masses of ρ, ϕ, D, D^*, and the condition $q_{i\alpha}^2-q_{j\alpha}^2=q_i^2=q_j^2$, we obtain the masses of $u_\alpha\approx d_\alpha, s_\alpha, c_\alpha$. To obtain the masses of u, d, s, and c we note that in the limit as $u, d\to 0$ we must have $m_\pi\to 0$, i.e., $m_\pi^2=2(u^2+d^2)$. An accurate calculation with allowance for the u–d splitting (from $\pi^+-\pi^0, K^+-K^0$) and for mixing has led to the final result

$$u = 0.063\quad d = 0.073,\ s = 0.337, c = 1.59\ \text{GeV}. \tag{5}$$

For these masses the relation $r_1\approx r^{2.11}$ is satisfied and θ_C is slightly larger than the value (4). If we neglect the u–d splitting, then the mass spectrum of the quarks satisfies the simple condition $[(u+d)/2]:s\sim(s/c)\sim r^2, r\approx 0.454$, which is quite close to the "extremal" value of r_0 in (4). We note that $r^2\sim\epsilon_\eta^2/m_k^2\sim g_E/g_D$ (see above). Other assumptions concerning weak corrections and concerning the spectrum of the masses of the quarks (for example, $r^1=r^{2+\epsilon}$) lead to different formulas for θ_C, but the simplest formula (4) agrees in best fashion with the value of θ_C obtained in experiment, and with the empirical masses of the quarks (5).

The author thanks V. A. Matveev for a valuable discussion and remarks.

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