Nonvortical phase slippage and oscillations of the vector l in He^3-*A*

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It is shown that the difference of the chemical potentials in He^3-*A* can be maintained by oscillations of the vector l in the absence of vortical motion. The force of friction between the normal and superfluid components turns out to be proportional to $(v^s-v^n)^3$.

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It is known from Anderson's work[1] that if one produces in the superfluid He II a chemical-potential difference between two points, then the phase difference of the condensate between these points increases continuously by virtue of the equality

$$\hbar\frac{\partial}{\partial t}(\phi_1-\phi_2)=\mu_1-\mu_2. \tag{1}$$

Since the superfluid velocity $\mathbf{v}^s$ is the gradient of the phase $\mathbf{v}^s=\hbar/m\,\nabla\phi$, the liquid between these two points should be accelerated. This acceleration, however, cannot persist continuously, and therefore, as shown by Anderson, a regime sets in wherein the line joining points 1 and 2 periodically crosses the quantized vortices. Each vortex decreases the phase difference by 2π. Therefore the number of vortices crossing this line per unit time is

$$\frac{\partial n_{\rm vort}}{\partial t}=\frac{1}{2\pi\hbar}(\mu_1-\mu_2). \tag{2}$$

In this regime the velocity $\mathbf{v}^s$ is on the average constant, i.e., the liquid flow is dissipative. The flux energy is consumed in overcoming the friction force produced in the viscous motion of the vortices.

In He^3-*A*, the order parameter is not the condensate phase ϕ, but a triad of unit vectors $\mathbf{\Delta}'$, $\mathbf{\Delta}''$, l. The superfluid velocity $\mathbf{v}^s$ is connected with the order parameter by the relation (see [2])

$$\mathbf{v}^s=\frac{\hbar}{2m}\Delta'_k\nabla\Delta''_k. \tag{3}$$

Which is the process that maintains the chemical-potential difference in He^3-*A* as does the vortex motion in He II? Anderson and Toulouse[3] proposed as one of the possible processes the motion of vortices that have no core. We shall show that another process is possible and is connected with the spatial and temporal oscillations of the vector l, wherein no vortices are produced, i.e., curl $\mathbf{v}^s=0$ everywhere.

To this end, we write down the hydrodynamic equations for He^3-*A* under the condition that the normal component is at rest. We neglect also the density variation, as well as all the dissipative processes with the exception of the dissipative transfer of the angular momentum from the internal motion of the liquid to the macroscopic flow

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of the liquid (the so called Cross rotational viscosity). These equations (see, e.g., [4,5]) take the form:

$$\frac{\partial \mathbf{v}^s}{\partial t} + \frac{\hbar}{2m} e_{ikl}\, l_i\, \vec{\nabla} l_k \frac{\partial l_l}{\partial t} = -\vec{\nabla}\mu, \tag{4}$$

$$\vec{\nabla}\mathbf{g} = 0, \tag{5}$$

$$\left[\mathbf{l},\ \frac{\partial \epsilon}{\partial \mathbf{l}} - \nabla_i \frac{\partial \epsilon}{\partial \nabla_i \mathbf{l}} + \frac{\hbar}{2m}(\mathbf{g}\,\vec{\nabla})\,\mathbf{l}\right] = -\gamma\left[\mathbf{l}\times\frac{\partial \mathbf{l}}{\partial t}\right]. \tag{6}$$

Here ϵ is the free energy of the liquid as a function of $\mathbf{l}$, $\nabla_i \mathbf{l}$, and $\mathbf{v}^s$, $g=\partial\epsilon/\partial\mathbf{v}^s$ is the flux of the liquid, and γ is the coefficient of the rotational Cross viscosity. Equation (4) replaces Eq. (1) for He II, (5) is the continuity equation, and (6) represents the balance of the moment of the forces, namely, the dynamic moment acting on the internal degrees of freedom of the liquid (left-hand side) is offset by the moment of the friction forces exerted by the macroscopic flow of the liquid (right-hand side).

We consider motion that depends on one coordinate z, along which $\mathbf{v}^s$ is directed. As seen from (4), an average nonzero chemical-potential gradient $\langle\partial\mu/\partial z\rangle$can be maintained by producing oscillations of the vector $\mathbf{l}$ in z and t such that

$$\frac{\hbar}{2m} \left\langle \mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial z}\times\frac{\partial \mathbf{l}}{\partial t}\right]\right\rangle = -\left\langle \frac{\partial \mu}{\partial z}\right\rangle. \tag{7}$$

As seen from the relation of Mermin and Ho[2], which follows from (3), we have here

$$(\mathrm{rot}\,\mathbf{v}^s)_i = \frac{\hbar}{4m} e_{ikl}\,\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial x_k}\times\frac{\partial \mathbf{l}}{\partial x_l}\right] = 0, \tag{8}$$

since $\mathbf{l}$ depends only on z. The characteristic periods z_0 and t_0 of the oscillations in z and t are obtained by comparing the different terms in Eq. (6)

$$z_0 \sim \frac{\hbar}{m\langle v^s\rangle}, \qquad t_0 \sim \frac{\gamma}{\rho^s\langle v^s\rangle^2}. \tag{9}$$

To ensure a nonzero mean value of the quantity

$$\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial t}\times\frac{\partial \mathbf{l}}{\partial z}\right],$$

we stipulate within each cell having a structure periodic in z and t a mapping of degree I of the cell area on the sphere $\mathbf{l}\cdot\mathbf{l}=1$ (region of variation of the vector $\mathbf{l}$) (see Fig. 1); the integral over the cell is then

$$\frac{1}{4\pi}\int_{\Delta S} dz\,dt\,\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial t}\times\frac{\partial \mathbf{l}}{\partial z}\right] = 1 = t_0 z_0 \left\langle \mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial z}\times\frac{\partial \mathbf{l}}{\partial t}\right]\right\rangle. \tag{10}$$

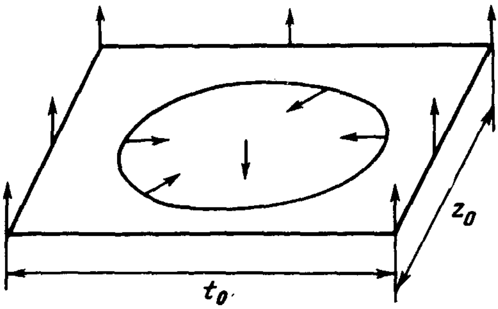

FIG. 1. Simplest form of the field $\mathbf{l}(z,t)$ in a cell of the structure periodic in z and t.

[An analogous phenomenon takes place in rotating He^3-A when the condition that the angular velocity of the rotation $\omega=(1/2)\langle \mathrm{curl}\, \mathbf{v}^2\rangle$ have a constant mean value calls for a nonzero mean value of

$$\mathbf{l}\left[\frac{\partial \mathbf{l}}{\partial x}\times\frac{\partial \mathbf{l}}{\partial y}\right],$$

as a result of which a structure periodic in the plane (x,y) of the vessel cross section is produced. Each cell of this structure is mapped on the sphere $\mathbf{l}\cdot\mathbf{l}=1$ with degree I].

From (10), (7), and (9) we obtain the connection between $\mathbf{v}^s$ and the average gradient of the chemical potential

$$<\frac{\partial\mu}{\partial z}>\sim\frac{\hbar}{mt_0 z_0}\sim\frac{\rho^s<v^s>^3}{\gamma}\quad . \tag{11}$$

Inasmuch as $\langle\partial\mu/\partial z\rangle$ has the meaning of the force of friction between the normal and the superfluid components, we obtain the same dependence of the friction force on v^s-v^n

$$F_{\mathrm{fr}}\sim(v^s-v^n)^3$$

as was experimentally observed in He II (see[7]).

It is possible that the experimentally observed[8] oscillations of the vector $\mathbf{l}$ are connected with excitation of the regime described above, wherein dissipative relative motion of the superfluid and normal components takes place, and to maintain this motion it is necessary to have a nonzero chemical-potential gradient. Another explanation of these oscillations advanced by Hall and Hook,[5] is connected with the formation of a solitary plane wave (soliton). The regime described above can, in particular, be represented as an assembly of such solitons. The dimension of each of the solitons is z_0. The solitons are interconnected in such a way that

$$<\mathbf{l}\left[\frac{\partial\mathbf{l}}{\partial z}\times\frac{\partial\mathbf{l}}{\partial t}\right]>$$

in each of them is of the same sign. When the solitons come in contact, the derivatives $\partial l_x/\partial z$ and $\partial l_y/\partial z$ can be discontinuous on the boundary between them. Whether such a boundary is stable and whether the solutions can be solved without a discontinuity of the derivatives is not yet clear.

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