Temperature of an electron-hole drop moving in a semiconductor

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The dependence of the electron-hole drop temperature on the drop velocity and on the lattice temperature is determined. It is shown that at subsonic velocities the drop is heated at high lattice temperatures and cooled at low lattice temperatures. At supersonic velocities, the drop is strongly heated.

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When an electron-hole drop moves under the influence of an external force, in addition to the appearance of friction with the lattice, the equilibrium between the phonon emission and absorption is shifted, as a result of which the drop temperature T_d can differ from the lattice temperature T.[1] This question was investigated in[2] for velocities $v<s$ (s is the speed of sound). According to[2] the temperature of a moving drop always exceeds the lattice temperature and increases monotonically with increasing v. We shall show that this conclusion is valid only at a sufficiently high lattice temperature, when $T_0/T=\xi<5.3$, where $kT_0=2p_F s$, and p_F is the Fermi momentum. At low lattice temperatures, however, when $\xi>5.3$, the moving drop is cooled. At supersonic velocities the drop should be strongly heated.

The change of the internal drop energy E as a result of the emission and absorption of acoustic phonons is determined by the expression[1] (for simplicitly we consider interaction with only one type of carrier)

$$\frac{dE}{dt}=-\frac{\pi D^2}{2\hbar\rho s}\int\frac{d^3\mathbf{p}d^3\mathbf{q}}{(2\pi\hbar)^6}q(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}-\mathbf{q}})\Big\{f_{\mathbf{p}}(1-f_{\mathbf{p}-\mathbf{q}})(N_{\mathbf{q}}+1)$$

$$-f_{\mathbf{p}-\mathbf{q}}(1-f_{\mathbf{p}})N_{\mathbf{q}}\Big\}\delta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}-\mathbf{q}}-sq+\mathbf{q}\mathbf{v}), \qquad (1)$$

where D is the deformation potential, ρ is the crystal density, f_p is the distribution function for the temperature T_d, and N_q is the equilibrium phonon distribution function for the lattice temperature T. The character of the change of the temperature of the moving drop can be explained in the following manner. Consider the quantity dE/dt at $T_d=T\ll T_0$. It is obvious that heating of the drop corresponds to $dE/dt>0$ and cooling to $dE/dt<0$. At $\beta=v/s<1$ it follows from the energy conservation law that $\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}-\mathbf{q}}=sq(1-\beta\cos\theta)>0$ (θ is the angle between q and v). Owing to the presence of degeneracy we have $\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}-\mathbf{q}}\sim kT$, so that at $\beta\ll 1$ the momenta of the emitted phonons are bounded: $q\sim kT/s\ll p_F$. This limitation causes expression (1), as well as the expression for the friction force,[1] to be proportional to the small factor $(T/T_0)^5$. With increasing velocity, the momenta of the phonons emitted at small angles to the velocity direction increase, whereas the momenta of the absorbed phonons remain as before at their thermal values. In view of the strong dependence of the integrand of (1) on $q(\sim q^4)$, this leads to predominance of emission over absorption, and consequently, to cooling of the drop. The reason for the cooling in the case of $T\ll T_0$ is most obvious at $\beta=1$. In this case in the angle region $\theta^2\lesssim T/T_0$ the restriction on the momentum of the emitted phonon is lifted so that $q\approx 2p_F$. The limitation on the interval of the angles θ turns out to be less significant than the increase of q. At supersonic velocities ($\beta>1$, $T\ll T_0$) the phonon emission turns out to be the predominant process, but it now leads not to cooling but to heating of the drop, inasmuch as the most substantial are transitions with increase of the internal energy, when $\beta\cos\theta>1$.[1)]

The equation that follows from (1) at $dE/dt=0$ for the stationary temperature T_d was obtained in[2]:

$$\int_{1-\beta}^{1+\beta} dt\, t^2 \int_0^{\xi} dz\, z^4 \frac{e^z - e^{zxt}}{(e^z-1)(e^{zxt}-1)}, \tag{2}$$

where $x=T/T_d$. By carrying out one integration we can recast (2) in a more convenient form

$$F(\xi_d(1+\beta),\xi) = F(\xi_d(1-\beta),\xi); \quad \xi_d = T_0/T_d; \tag{3}$$

$$F(\gamma,\xi) = \frac{1}{\gamma^2}\int_0^{\gamma}\frac{(\gamma z)^2 - z^4}{e^z-1}dz - \frac{2\gamma^3}{3\xi^5}\int_0^{\xi}\frac{z^4 dz}{e^z-1}. \tag{4}$$

Let us determine the dependence of the drop temperature T_d on the lattice temperature T and on the drop velocity v in limiting cases. At $\xi\ll 1$ and $\xi_d\ll 1$ we expand the function F in a series and obtain from (3) the expression

$$T_d = T(1+\beta^2/3), \tag{5}$$

At low velocities ($\beta\ll 1$), the change of the drop temperature is of the order of β^2[1]:

$$T_d = T\,[1 + G(\xi)\,\beta^2], \tag{6}$$

An explicit expression for the function G can be obtained by expanding the function F in powers of β up to third-order terms. We then obtain $G(\xi)=I_1(\xi)/I_2(\xi)$, where

$$I_1(\xi) = \frac{1}{6}\int_0^{\xi}\frac{(4-z)e^{2z}-(z+4)e^z}{(e^z-1)^3}z^5 dz; \quad I_2(\xi) = \int_0^{\xi}\frac{e^z z^5 dz}{(e^z-1)^2}. \tag{7}$$

The function $G(\xi)$ reverses sign at $\xi=5.3$, $G(0)=1/3$, $G(\infty)=-1/3$. Thus, at low velocities and at $T=0.19T_0$ the heating of the drop gives way to cooling.

We turn now to the case of high velocities and low temperatures ($\xi \gg 1$). At $\beta=1$ Eq. (3) takes the form $F(2\xi_d,\xi)=0$. Using the asymptotic values of the integrals in (4), we obtain for this case

$$T_d = 3.8\ T(T/T_0)^{2/3}. \tag{8}$$

At sufficiently low temperatures $T<0.14T_0(\xi>7)$ we have $T_d<T$. The result (8) is evidence that in principle deep cooling of the drop is possible at very low temperatures and at $\beta \approx 1$.

At supersonic velocities the drop is heated, and if $T \ll T_0$ then the drop temperature does not depend on the lattice temperature (in contrast to the case $\beta<1$). From (3) and (4) we get

$$T_d = 0{,}42\ T_0(\beta-1) \qquad \text{at} \quad \xi^{-5/3} \ll \beta-1 \ll 1; \tag{9a}$$

$$T_d = (2/15)\ T_0\beta^2 \qquad \text{at} \quad \beta \gg 1. \tag{9b}$$

Figure 1 shows plots, obtained by numerically solving Eq. (3), of T_d against β at different lattice temperatures T. Formula (5) agrees within 5% of with the exact

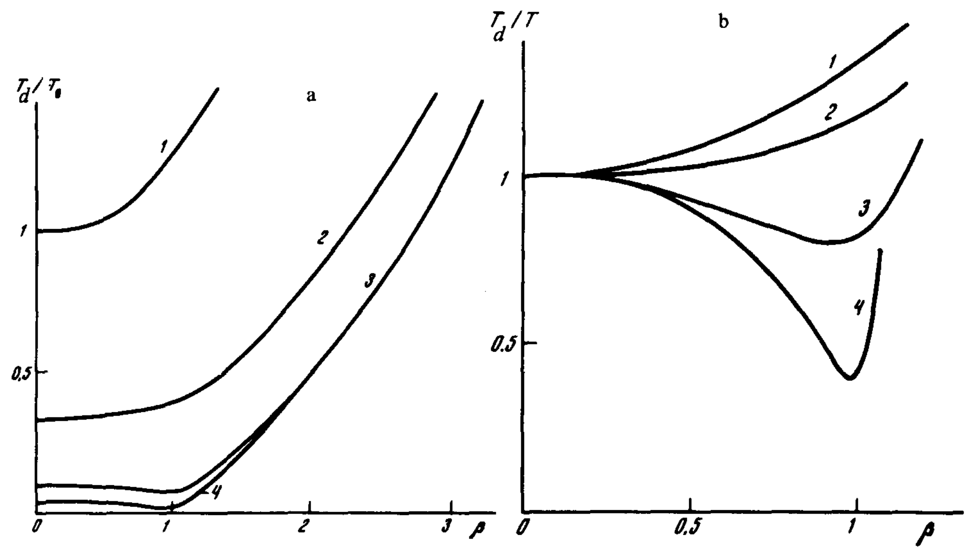

FIG. 1. Dependence of the drop temperature T_d on the drop velocity d at different lattice temperatures T: a—drop temperature in units of T_0, b—the ratio T_d/T. Values of the parameter $\xi=T/T_0$: 1–1, 2–3, 3–10, 4–30; $\beta=v/s$.

solution up to $T=T_0$. Formulas (9a) and (9b) are close to the exact solution at $\beta \leqslant 2$ and $\beta \geqslant 2$, respectively.

For germanium, $T_0 \approx 13$ K and the temperature at which the heating gives way to cooling is ≈ 2.5 K. At not too low temperatures, however, the changes in the drop temperature at subsonic velocities are small. Thus, at $T=1$ K and $v=s$ we should have $T_d \approx 0.7$ K. On the contrary, at supersonic velocities the drop becomes strongly heated and may evaporate.

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[1)]At $\beta > 1$, the drop, acting as a macroscopic body, begins to emit Cerenkov radiation of long-wave phonons. This radiation must be taken into account when the deceleration force is determined, but it does not change the internal energy of the drop and is of no importance for the determination of T_d at a given velocity v.

[1]L.V. Keldysh, in: Éksitony v poluprovodnikakh (Excitons in Semiconductors), Nauka, 1971, p. 5.
[2]S.G. Tikhodeev, Kratkie soobshcheniya po fizike, No. 5, 13 (1975)