Spin glass with short-range action in the vicinity of the "transition"

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The behavior of spin glass with nonmagnetic defects in the vicinity of the percolation threshold in a finite magnetic field (h) is analyzed by percolation-theory methods. It is shown that the heat-capacity increment $\Delta C(h) = C(h) - C(0)$ and the magnetic susceptibility $\partial M/\partial h$ are directly connected with the cluster dimension distribution function. Power-law dependences on h and $(T-\Theta)/\Theta$ are obtained on the basis of similarity theory.

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It was shown in a preceding article[1] that for spin glass with a large number of defects, in which the interaction of the spins depends exponentially on the distance, it is possible to calculate all the principal thermodynamic and kinetic characteristics. In the "transition" region, where the temperature is of the order of the spin interaction at an average distance $n_m^{-1/3}$ (n_m is the concentration of the magnetic atoms), the percolation approach proposed for spin glasses in[2] was used. It followed from this approach that the heat capacity and the resistance have in fact no singularities at the percolation-threshold point, and the magnetic susceptibility, as a function of T, can have a small "beak." Thus, percolation effects manifest themselves quite weakly.

It will be shown in the present article that the behavior of the heat capacity and of the magnetic susceptibility in the vicinity of the transition in a finite magnetic field, as functions of the temperature and of the magnetic field are directly connected with the principal characteristics of the percolation problem.

As already stated in[11], in the vicinity of the "transition" $T=\Theta$ or $\beta=\beta_c$ [here $\beta=(4\pi/3)n_m r^3(T), \beta_c=3.0\pm0.1$; $r(T)$ is the "thermal radius"] there are produced large clusters of magnetic whose spin rotates as a unit. In an external magnetic field h, the heat capacity connected with the spin flip is

$$C_m = \phi\left(\frac{1}{2}\ \frac{\mu h}{T}\right) - \phi\left[\left(Sm + \frac{1}{2}\right)\frac{\mu h}{T}\right], \tag{1}$$

where $\phi(x)=x^2/\sinh^2 x$, S is the spin of an individual particle and Sm is the spin of the cluster, with the assumption $m\gg1$. We are interested in the field region where $Sm\mu h/T\sim1$, and the first term in (1) can be replaced by unity. In addition, individual weakly bound spins contribute to the heat capacity (see[11]). At $\mu hS/T\ll1$, however, this contribution depends little on the magnetic field. Therefore the sum of C_m over all the clusters is $\Delta C(h)=C(h)-C(0)$. It is easy to verify that the following relation holds

$$\frac{\partial Mm}{\partial h} = \frac{T}{h^2}\ C_m, \tag{2}$$

where $M_m=\mu SmB_{Sm}(\mu h/T)$ is the magnetic moment of the cluster and B_{Sm} is the Brillouin function. Consequently, the same relation holds also for the summed quantities.

We shall sum over the clusters in two stages. We sum first over all m at a given number n of the particles in the cluster. To this end we must find the probability of the given total spin in a cluster of n particles. So long as the linear dimension of the cluster does not exceed $r_1=(4\pi n_m l)^{-1/2}$, all the spins in the cluster are collinear (see[11]). The probability that $n\uparrow$ spins have an "up" orientation and the remainder $n-n\uparrow$ have a "down" orientation is equal to $(\frac{1}{2})^n n!/[n\uparrow!(n-n\uparrow)!]$. Using the asymptotic expression for large n and $n\downarrow$, and recalculating into the probability relative to $m=|n\uparrow-n\downarrow|$, we obtain

$$B_1(n, m)\ dm = \sqrt{2/\pi n}\exp(-m^2/2n)\ dm. \tag{3}$$

If the linear dimension of the cluster exceeds r_1, i.e., the number of particles in it exceeds $n_1=(1/3)(4\pi n_m l^3)^{-1/2}$, then the cluster becomes "multidomain," i.e., it consists of individual sections of n_1 particles, in each of which the spins are collinear, but the spins of the different sections have different orientations. The function $B(n,m)$ can be obtained by assuming a model in which independent averaging over the orientation of the different sections is possible. This corresponds apparently to the assumption that the interaction of a given section with the neighbors is via a large number of boundary spins. Assuming an isotropic angular distribution for the spin orientation of each section, we obtain in place of B_1 the distribution

$$B_3(n, m) = \sqrt{2/\pi}\,(3/n)^{3/2}\, m^2 \exp\left(-\frac{3}{2}\ \frac{m^2}{n}\right). \tag{4}$$

In this manner we can find $B(n,m)$ for any angular distribution, and it always turns out that

$$\overline{m}^2 = \int_0^\infty m^2 B(n,m)\, dm = n. \quad (5)$$

Averaging C_m in $B_1(n,m)$ we have

$$C_{n1} = \int_0^\infty C_m B_1(n,m)\, dm = \sqrt{2/\pi\rho} \int_0^\infty \exp(-y^2/2\rho)\,[1-(y/\mathrm{sh}\,y)^2]\,dy, \quad (6)$$

where $\rho = n\alpha^2$, $\alpha = \mu hS/T$. The limiting values take the form

$$C_{n1} \approx \rho/3 - \rho^2/5, \qquad \rho \ll 1 \quad (7)$$

$$C_{n1} \approx 1 - \frac{\pi^{3/2}}{3\sqrt{2}}\rho^{-1/2} + \frac{\pi^{7/2}}{2^{3/2}\cdot 15}\rho^{-3/2}, \qquad \rho \gg 1. \quad (8)$$

For the distribution $B_3(n,m)$ we obtain

$$C_{n3} = \sqrt{2/\pi}\,(3/\rho)^{3/2} \int_0^\infty y^2 \exp(-3y^2/2\rho)\,[1-(y/\mathrm{sh}\,y)^2]\,dy, \quad (9)$$

$$Cn_3 \approx \rho/3 - \rho^2/9, \qquad \rho \ll 1 \quad (10)$$

$$C_{n3} \approx 1 - \frac{\pi^{7/2}}{5}\left(\frac{3}{2}\right)^{1/2}\rho^{-3/2} + \frac{\pi^{11/2}}{7}\left(\frac{3}{2}\right)^{3/2}\rho^{-5/2}, \quad \rho \gg 1. \quad (11)$$

To obtain the total magnetic heat capacity it is necessary to know the distribution $A(n,\epsilon)$ of the finite clusters with respect to the dimensions, where $\epsilon = (\beta_c - \beta)/\beta_c$. This function is higher than the percolation threshold, i.e., at $\epsilon > 0$ it should satisfy the normalization condition:

$$\sum_{n=1}^\infty n\, A(n,\epsilon) = N, \quad (12)$$

here N is the total number of particles or

$$\sum_{n=1}^\infty n\,[A(n,\epsilon) - A(n,0)] = 0. \quad (13)$$

It follows from similarity theory (see[3]) that an $n \gg 1$ we have

$$A(n,\epsilon) \approx N n^{-b} f(\epsilon n^a), \quad (14)$$

where a and b are certain constants, with $2 < b < 3$ and $f(0) = \mathrm{const}$. According to computer calculations, $f(x)$ decreases exponentially at $|x| \gg 1$.

Below the transition point the sum (12) does not include the particles of the infinite cluster. Therefore

$$\sum_{n=1}^{\infty} nA(n,\epsilon) = N[1-\Theta(-\epsilon)P(|\epsilon|)], \tag{15}$$

where $P(|\epsilon|)$ is the probability that the particle is in an infinite cluster. The average number of particles in the finite cluster is

$$R(\epsilon) = \frac{1}{N}\sum_{n=1}^{\infty} n^2 A(n,\epsilon). \tag{16}$$

Substituting (14), we get

$$P(\epsilon) \sim |\epsilon|^{\xi}, \quad \xi = (b-2)/a,$$
$$R(\epsilon) \sim |\epsilon|^{\gamma}, \quad \gamma = (3-b)/a. \tag{17}$$

From the numerical calculations (see[4]) it follows that $\xi=0.35$, $\gamma=1.69$, and consequently $a=0.49$, $b=2.17$.

The magnetic heat capacity per unit volume is

$$\Delta C(h) = \frac{1}{V}\sum_{n=1}^{\infty} A(n,\epsilon)\, C_n. \tag{18}$$

Since $b>2$ in (14), this sum is accumulated principally at $n\sim 1$, where the asymptotic form of C_n at $\rho \ll 1$ can be used. It should be noted that for a one-domain cluster the condition (5) is satisfied at all n. Comparing with (12), we get

$$\Delta C(h) = \Delta C^{(1)} + \Delta C^{(2)}, \tag{19}$$

$$\Delta C^{(1)} = \frac{1}{3} n_m a^2[1-\Theta(-\epsilon)P(|\epsilon|)], \tag{20}$$

$$\Delta C^{(2)} = n_m \int_0^{\infty} n^{-b}\left(C_n - \frac{1}{3}na^2\right) f(n^a\epsilon)\, dn. \tag{21}$$

The integral for $\Delta C^{(2)}$ extends over large n, thus justifying the use of formula (14). Recognizing that C_n depends on the combination $n\alpha^2$, and $P(\epsilon)$ takes the form (17), we obtain the general similarity relation

$$\Delta C(h) - \frac{1}{3} n_m a^2 = a^{2(b-1)}\phi(\epsilon a^{-2a}). \tag{22}$$

In the case $|\epsilon| \gg \alpha^{2a}$ we obtain from the asymptotic forms of (7) and (9)

$$\Delta C^{(2)} \approx -\frac{-a^4}{5} n_m R(\epsilon) \quad |\epsilon| >> n_1^{-a}, \tag{23}$$

$$\Delta C^{(2)} \approx -\frac{-a^4}{9} n_m R(\epsilon) \quad |\epsilon| << n_1^{-a}. \tag{24}$$

We note that this dependence on ϵ is the principal one at $\epsilon>0$, but at $\epsilon<0$ the term with $P(\epsilon)$ in $\Delta C^{(1)}$ predominates. In the vicinity of $\epsilon=0$, i.e., at $|\epsilon| \ll \alpha^{2a}$, we have

$$\Delta C^{(2)} \rightsquigarrow -\alpha^{2(b-1)}[1 + q\epsilon\alpha^{-2a}], \tag{25}$$

where $q > 0$ and $q \sim 1$.

Since, $(\beta_c - \beta)/\beta_c \sim (n_m^{1/3} l)(T - \Theta)/\Theta$, the temperature dependence of the terms with ϵ in $\Delta C(h)$ is weaker than the dependence that stems from the principal term $(1/3)n_m\alpha^2$. It is better therefore to determine in the experiment $\partial^2 M/\partial h^2$. Comparing with (2), we see that the terms corresponding to $\Delta C^{(1)}$ in (2) are excluded from this quantity.

More detailed calculations are made for two model functions $A(n,\epsilon)$: a) the function $A(n,\epsilon) = \sqrt{2\pi}\, n^{5/2})^{-1}\exp(-n\epsilon^2/2)$, obtained for a Bethe lattice, and b) the function $A(n,\epsilon)$ in the form (14), where $f(x) = D(1 + xs/\xi)\exp(-sx)$ at $x > 0$ and $f(x) = D\exp[s(\xi^{-1} - 1)x]$ at $x < 0$, where s and d are fitting constants. The results will be reported in a more detailed paper.

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