Influence of electron-electron correlations on the resistivity of dirty metals

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(Submitted 8 May 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 12, 700–702 (20 June 1978)

We investigate the influence of the effective electron-electron interaction on the temperature-dependent part of the resistivity. It is shown that the sign of the increment is determined by the sign of the interaction constant, and the absolute value is $\Delta\rho \sim T^{1/2}$. In the case of repulsion $\Delta\rho < 0$ and therefore the temperature dependence of the resistivity has a minimum.

PACS numbers: 72.10.Fk

It is well known that if umklapp processes are not taken into account then the electron-electron collisions make no contribution to the resistivity of metals, since they conserve the total momentum. However, as will be shown in the present article, in dirty metals, owing to interference with scattering by the impurities, the electron-electron interaction makes a finite temperature-dependent contribution to the resistivity: at low temperatures, such that $T \ll \hbar/\tau$,

$$\frac{\Delta\rho}{\rho_0} \approx -0.4\,\frac{4\pi e^2}{\kappa^2}\,\nu(\mu)\,\frac{T^{1/2}\hbar^{3/2}}{\mu^2\tau^{3/2}}, \tag{1}$$

where T is the temperature, τ is the time of the momentum relaxation on the impurities, μ is the chemical potential, ρ_0 is the residual resistivity, $\nu(\mu)$ is the density of states on the Fermi surface, and κ^{-1} is the Debye screening radius. For the free-electron model $(4\pi e^2/\kappa^2)\nu(\mu)=1$.

One of us[1] has shown that allowance for the interference of scattering by phonons and impurities, leads, at sufficiently low temperatures, to an expression that differs from (1) in that $-4\pi e^2/\kappa^2$ is replaced by g^2—the square of the electron-phonon interaction constant.

Thus, expression (1), following replacement of $(-4\pi e^2/\kappa^2)\nu(\mu)$ by the dimensionless constant λ of the effective electron-electron interaction, is valid for any type of electron-electron interaction. Therefore the sign of the correction depends on the sign of the interaction constant: the effective attraction leads to an increase of the resistivity with temperature, while the effective repulsion leads to its decrease, and consequently, to the appearance of a minimum on the plot of $\Delta\rho$ against the temperature.

Since the square-root dependence on the temperature is the slowest of the presently known dependences, it can be stated that at sufficiently low temperatures the temperature-dependent part of the resistivity is proprotional to $T^{1/2}$.

According to[1], the relative change of the resistivity is of the form

$$\frac{\Delta\rho}{\rho_0} = \frac{1}{v_F^2}\,\mathrm{Im}\int_0^\infty \frac{d\omega}{\pi}\int\frac{d^3q}{(2\pi)^3}\left(\frac{\mathrm{sh}\,\frac{\omega}{T}-\frac{\omega}{T}}{\mathrm{ch}\,\frac{\omega}{T}-1}-1\right)|g(q)|^2 \times D^r(q,\omega)\,J(q,\omega), \tag{2}$$

where the function $J(q,\omega)$ at $qv_F\tau \ll 1$ and $\omega\tau \ll 1$ is equal to

$$J(q,\omega) = \frac{4}{3}\,v_F^2\,\frac{Dq^2}{(-i\omega + Dq^2)^3} \tag{3}$$

$D=(1/3)v_F^2\tau$ is the electron diffusion coefficient, $D^r(q,\omega)$ is the retarded Green's function of the boson, $g(q)$ is the constant of the interaction of the electron with this Bose excitation. In the case of direct electron-electron interaction we have

$$|g(q)|^2 D^r(q,\omega) = \frac{4\pi e^2}{q^2\epsilon(\omega,q)} \tag{4}$$

where $\epsilon(\omega,q)=1+D\kappa^2/(-i\omega+Dq^2)$ is the permittivity in the low-frequency and long-wave limit. The important quantities in (2) are $q\sim(\omega/D)^{1/2}$ and $\omega\sim T$. Therefore formulas (3) and (4) are valid at $T\tau \ll \hbar$. Substituting (3) and (4) in (2) we obtain

$$\frac{\Delta\rho}{\rho_0} = c\,\frac{\sqrt{6}}{16}\,\frac{T^{1/2}\,\hbar^{3/2}}{\mu^2\,\tau^{3/2}}\,,$$

$$c = \int_0^\infty dx\left(\frac{\operatorname{sh} x - x}{\operatorname{ch} x - 1} - 1\right) \approx -2.5\,. \tag{5}$$

In our opinion, the effect is connected with the electron correlations. Effective attraction decreases the resistivity, and when the temperature rises the correlation becomes weaker, and it is this which causes the resistivity to increase with temperature. In the case of repulsion, the correlations increase the resistivity, therefore $\partial\rho/\partial T < 0$.

Let us estimate $\Delta\rho/\rho_0$, say for vanadium. For samples with $R_{300\,\mathrm{K}}/R_{4.2\,\mathrm{K}} \approx 1$ the mean free path is $l_{\mathrm{Im}} \approx 3\times10^{-7}$ cm. Since $\mu\sim0.9$ eV and $v_F \approx 1.7\times10^{-7}$ cm^{-7} $\mathrm{cm/sec}^{-1}$, we have $\hbar/\tau \approx 400$ K and

$$\frac{\Delta\rho}{\rho_0} \approx 6.4\times10^{-4}\left(\frac{T}{400}\right)^{1/2}.$$

The absolute change in the resistivity is $\Delta\rho\sim10^{-8}$–10^{-9} Ω-cm, and can therefore be readily measured. It is clear that it is desirable to choose for the observation semimetals or degenerate semiconductors with small Fermi energies, where this effect is much larger. For example, for tellurium with hole density $n\sim10^{18}$ cm^{-3} we have $\mu\sim120$ K, $\hbar/\tau\approx60$ K, therefore $\Delta\rho/\rho_0\sim0.1\,(T/60)^{1/2}$.

[1]B.L. Al'tshuler, Zh. Eksp. Teor. Fiz. **75**, No. 3 (1978) [Sov. Phys. JETP **48**, No. 3 (1978)].