# Weak ferromagnetism of the $A$ phase and singularities of the superfluid phase transition in $He^3$

É. B. Sonin

*A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences*
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The dipole-dipole interaction in superfluid $He^3$ is the source of the coupling between the orbital and spin angular momenta of the Cooper pair (**LS** coupling). As a result, the second-order phase transition splits into two closely located transitions of second and first order, and a resultant nuclear spin appears in the $A$ phase.

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Recently Paulson and Wheatley[1] observed experimentally a small spontaneous magnetic moment in the $A$ phase of superfluid $He^3$. A weak electronic ferromagnetism of the $A$ phase was predicted theoretically by Leggett.[2] We shall show below that in the $A$ phase there should exist also a nuclear spin magnetic moment due to dipole-dipole interaction. It does not conserve the orbital and spin angular momenta separately, but conserves their sum $\mathbf{J}=\mathbf{L}+\mathbf{S}$, i.e., it leads to appearance of **LS** coupling. This coupling influences the character of the phase transition from the normal to the superfluid state, inasmuch as the energy of the superfluid phase and hence also the critical temperature, becomes dependent on the total angular momentum of the motion $J$.

In general form, the **LS**-coupling energy quadratic in the order parameter is given by

$$\mathcal{H}_{LS} = a_1 d^*_{ii} d_{jj} + a_2 d^*_{ij} d_{ji} . \tag{1}$$

The first index of the order-parameter matrix $d_{ij}$ pertains to the orbital space, and the second to spin space. Normalizing $d_{ij}$ to the gap $\Delta$, i.e., $d^*_{ij} d_{ij} = \Delta^2$, we obtain for the matrices $d_{ij}$, which are the eigenfunctions of the total angular momentum $J$ of the Cooper pair:

$$\begin{aligned} \mathcal{H}_{LS} &= (3a_1 + a_2)\Delta^2 && \text{at} \quad J = 0, \\ \mathcal{H}_{LS} &= -a_2\Delta^2 && \text{at} \quad J = 1, \\ \mathcal{H}_{LS} &= a_2\Delta^2 && \text{at} \quad J = 2. \end{aligned} \tag{2}$$

If $a_2 < -(3/2)a_1$ and $a_1 < 0$, then the lowest energy $\mathcal{H}_{LS}$ and the largest critical temperature correspond to $j=0$, i.e., to the $B$ phase with matrix $d_{ij} = (\Delta/\sqrt{d})\delta_{ij}$. This is precisely the phase which appears on the second-order phase transition line. If $a_1 > 0$ and $a_2 < 0$, then the lowest energy is possessed by the phase with $J=2$ (symmetric $d_{ij}$ matrix with zero trace). At $a_2 > 0$ and $a_2 > -(3/2)a_1$, the phase with $J=1$ is energywise favored (antisymmetrical matrix $d_{ij}$), which we shall call the vector phase, since an antisymmetrical $3\times 3$ matrix can be expressed in terms of the components of a vector. The $A$ phase, which is a combination of phases with $J=1$ and $J=2$, never appears on the second-order phase transition line. If the LS coupling is due to dipole-dipole interaction for which $a_1=a_2$,[3] then a vector phase appears on the second-order transition line. However, if the temperature is even slightly lowered, the decisive role in the form of the wave function of the Cooper pair is assumed by the anharmonic terms of the Ginzburg-Landau expansion, and a first-order phase transition takes place to the $B$ phase or to the weakly ferromagnetic $A$ phase which differs from the hitherto known nonferromagnetic $A$ phase (the Anderson-Borel-Brinkman phase) in the presence of a summary small nuclear pair spin parallel to its orbital angular momentum.

To describe the indicated two phase transitions with ultimate formation of the weakly ferromagnetic $A$ phase, we use the Ginzburg-Landau expansion, which takes into account both the effect of the strong coupling in paramagnon exchange and the dipole-dipole interaction[3]

$$F=\frac{1}{2}\frac{\partial n}{\partial\epsilon}\left\{-\tau d^*_{ai}d_{ai}+G(d^*_{ii}d_{jj}+d^*_{ij}d_{ji})+\frac{3}{10}\bar{\beta}\left[-|d_{ai}d_{ai}|^2\right.\right.$$

$$+2d^*_{ai}d^*_{aj}d_{\beta i}d_{\beta j}-(2+\delta)d^*_{ai}d^*_{\beta i}d_{aj}d_{\beta j}+(2+\delta)(d^*_{ai}d_{ai})^2$$

$$\left.\left.+(2-\delta)d^*_{ai}d^*_{\beta j}d_{aj}d_{\beta i}\right]\right\}, \qquad (3)$$

where $\tau=1-T/T_c$, $T_c$ is the critical temperature without allowance for the dipole-dipole interaction due to the terms of (3) that are linear in the constant $G$, and $\partial n/\partial\epsilon$ is the state density. The constants $\bar{\beta}$ and $G$ are equal to

$$\bar{\beta}=\frac{7}{8}\xi(3)(\pi kT_c)^{-2};\qquad G_0=\frac{18}{25}\frac{(2-\delta)g_0\bar{\beta}}{\frac{\partial n}{\partial\epsilon}},$$

where, according to Leggett,[3] $g_0=10^{-3}$ erg/cm$^3$.

The $A$ phase is energywise more favored than the $B$ phase when the constant $\delta>\frac{1}{4}$.[3] at $\delta=1/4$ we have $G=0.45\times 10^{-5}$.

We seek the matrix $d_{ij}$ in the form

$$\hat{d}=\frac{\Delta}{[(1+|\alpha|^2)(1+|\beta|^2)]^{1/2}}\begin{pmatrix}0 & -\alpha & -i\alpha\beta\\ 1 & 0 & 0\\ i\beta & 0 & 0\end{pmatrix}. \qquad (4)$$

The nonferromagnetic $A$ phase corresponds to the values $\alpha=0$ and $\beta=1$ in (4), while the vector phase corresponds to $\alpha=1$.

Substituting (4) in (3) we get

$$F = \frac{1}{2}\frac{\partial n}{\partial \epsilon}\left\{-\Delta^2\left[\tau + G\frac{a + a^*}{1 + |a|^2}\right] + \frac{3}{10}\bar{\beta}\Delta^4\frac{1}{(1 + |a|^2)^2}\right.$$

$$\left.\times\left[2 - \delta + 2(2 + \delta)|a|^2 + 6|a|^4 + \frac{|1 - \beta^2|^2}{(1 + |\beta|^2)^2}(1 - a^2 - a^{*2} - (3 + \delta)|a|^4)\right]\right\}. \tag{5}$$

Obviously, any minimum of the free energy (5) corresponds always to real and non-negative $\alpha$, and also to imaginary $\beta$, if $1-2\alpha^2-(3+\delta)\alpha^4<0$, or to $\beta=1$ if $1-2\alpha^2-(3+\delta)\alpha^4>0$. In the former case the minimum of $F$ is realized if $\alpha=1$ and corresponds to a vector phase with energy

$$F_V = -\frac{5}{12}\frac{\partial n}{\partial \epsilon}\bar{\beta}^{-1}(\tau + G)^2. \tag{6}$$

A second-order phase transition to this phase occurs at $\tau=-G$.

The phase with $\beta=1$ is a weakly ferromagnetic $A$ phase with energy

$$F_A = -\frac{5}{6}\frac{\partial n}{\partial \epsilon}\frac{1}{\bar{\beta}}\frac{[\tau(1 + a^2) + 2Ga]^2}{2 - \delta + 2(2 + \delta)a^2 + 6a^4}. \tag{7}$$

The energy $F_A$ is minimal at a value $\alpha$ satisfying the equation

$$\frac{\tau}{G} = \frac{2 - \delta - 6a^4}{(2\delta a + (4 - \delta)a^3}. \tag{8}$$

At $\tau=\tau_1$, where the condition $F_V=F_A$ is satisfied, a first-order phase transition from the vector phase into the $A$ phase takes place. If $\delta=\frac{1}{4}$, then $\tau_1\approx 156$ and $\alpha\approx 0.19$ at the transition point. In final analysis the temperature interval between the second- and first-order transition points is small and equals $\Delta T=T_c(\tau_1+G)\approx 0.18\times 10^{-3}$ mK.

In the vector phase there is no magnetic moment. We determine the spontaneous magnetic moment in the weakly ferromagnetic $A$ phase by using for the spin density an expression obtained by Fomin[4]

$$M = 5\frac{\gamma a}{\bar{\beta}}\frac{a^2[\tau(1 + a^2) + 2Ga]}{2 - \delta + 2(2 + \delta)a^2 + 6a^4} \underset{\tau \gg G}{\rightarrow} \frac{5(2 - \delta)}{4\delta^2}\frac{\gamma a}{\bar{\beta}}\frac{G^2}{\tau}$$

$$\approx 0.2\times 10^{-13}\tau^{-1}\ \mathrm{G}, \tag{9}$$

where $\gamma$ is the nuclear gyromagnetic ratio, $a=2.5\times 10^{27}$ sec/erg-cm$^3$ is the constant determined by Fomin from the experimental data.

At the first-order transition point the moment turned out to be $0.2\times 10^{-9}$ G. Experiment[1] yielded momenta of the same order, but in a region quite remote from the phase transition region where the nuclear angular moment is much lower than the electron spin, estimated by Leggett,[2] which increases with temperature like $\tau$. Therefore the nuclear angular momentum can be detected in measurements closer to the critical temperature, and this would make it possible to observe on the first-order transition line a jump of the nuclear angular momentum which greatly exceeds the jump of the electron spin.

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