Convective transport in a turbulent plasma

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It is shown that the interaction of convective cells during the nonlinear stage of development of instability in a plasma situated in a magnetic field with shear leads to the formation of ordered turbulent convective streams. Kinetic equations describing the turbulent-convective transport are obtained.

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The development of current-convective and drift instabilities during the nonlinear stage leads to formation of convective cells in a plasma situated in a magnetic field,[1,2] a fact that exerts a substantial influence on the transport across the magnetic field. The purpose of the present article is to show that the interaction of the convective cells

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leads to their ordering and to the formation of turbulent convective streams transverse to the magnetic field, which are typical of the moderately turbulent state of the plasma.[1)]

We consider for the sake of argument the development of current-convective instability in a homogeneous plasma situated in a toroidal magnetic field with shear in the presence of a temperature gradient $\partial T/\partial r < 0$. As shown in[1], in the vicinity of the resonant magnetic surfaces there are produced during the nonlinear stage convective cells with characteristic dimensions, velocity, and convection time

$$x_m = r\xi^{1/3} m^{-2/3}, \qquad v_m = 2A\widetilde{T}/x_m, \qquad \tau_m = x_m/v_m, \tag{1}$$

where

$$\xi = A|\partial T/\partial r|r^{-3}a^{-1}, \qquad A = qcE_0(d\sigma/dT)/q'H_\theta\sigma, \quad a = \chi_{\parallel}\theta^2 r^{-4}$$

$\widetilde{T}$ is the temperature perturbation, q is a margin coefficient, c is the speed of light, E_0 is the intensity of the longitudinal electric field, H_θ is the intensity of the poloidal magnetic field, σ is the conductivity, θ is the shear, and $\chi_{\parallel}$ is the coefficient of longitudinal thermal conductivity. The pair of indices (m,n) number the resonant surfaces. The average linear instability growth rate is equal to

$$\gamma_m \approx -2A(\partial T/\partial r)x_m^{-1}. \tag{2}$$

The characteristic distance between the resonant surfaces with cells of comparable scale is[1] $\Delta_m \approx q^2/q'm^2$. Since only the very largest cells make the main contribution to the transport, provided that they overlap, it follows that by equating x_m and Δ_m we find that the principal scale of the cells is determined by the number $m_0 \approx [q^2/q'r\xi^{1/3}]^{3/4}$.

During the initial stage of the instability, near the resonant surfaces, convection develops and is enhanced by the average temperature gradient, and the streams that develop as a result of the initial fluctuations on different surfaces are not correlated with one another. During the nonlinear stage, when the convective streams alter the temperature distribution significantly, the overlapping cells influence one another strongly. During this stage, spatial ordering of the cells take place and the convective streams are produced (see Fig. 1). This ordering is favored: the instability increases,

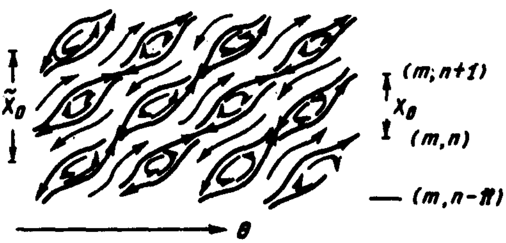

FIG. 1.

since the temperature gradient increases locally and on the other hand the ordering of the convective streams increases the heat transport [see (5) and (6)].

In addition to cells of the main scale, small-scale cells with $m_0 < m < m_c$ (m_c is the smallest instability scale and is determined by the thermal conductivity, see[1]) are unstable and turbulize the convective stream.

The kinetic equations of the moderate turbulence can be represented in the form:

$$T_1(r,t) = w_0 T_1(r - x_0, t - \tilde{\tau}_0) + (1 - w_0) T_2(r, t - \tilde{\tau}_0),$$

$$T_2(r,t) = (1 - w_0) T_1(r, t - \tilde{\tau}_0) + w_0 T_2(r + x_0, t - \tilde{\tau}_0), \tag{3}$$

where T_1 and T_2 is the average temperature in the ascending and descending streams, $\tilde{\tau}_0 = \sqrt{2x_0/v_0}$ and $\tilde{v}_0$ are defined by relations (1) in which we must put $m = m_0$, $T = T_+ = \frac{1}{2}(T_1 + T_2)$, $\tilde{T} = T_- \equiv \frac{1}{2}(T_1 - T_2) w_0$ is the ordering coefficient, which describes the probability that the flux element will remain in the ascending (descending) stream after going around the convective cell. The transition of an element from one stream to another is due to turbulent transverse transport, and therefore

$$w_0 \approx \frac{1}{2}\left[1 + \exp\left(-\frac{\pi^2 \chi_{\perp 0} \tilde{\tau}_0}{x_0^2}\right)\right]. \tag{4}$$

The expression for the coefficient $\chi_{\perp 0}$ of the transverse turbulent thermal conductivity can be found in[1] with allowance for the fact that only small cells contribute to it.

Equations (3) coincide in form with the equations of the moderately turbulent relaxation of a beam that is smeared in velocity.[3] It follows from them that in the quasistationary state we have

$$T_- = w_0 x_0 T_+' / 2(1 - w_0), \qquad T_+' \equiv \partial T/\partial r, \tag{5}$$

$$q_T = \frac{1}{4} [w_0/(1 - w_0)]^2 x_0^2 \gamma_0 T_+'; \qquad \gamma_0 \equiv \gamma_{m_0}. \tag{6}$$

Here q_T is the heat flux.

It is seen from (5) that in the region of localization of the cell the temperature gradient $\sim 2T_-/x_0$ is larger by a factor $w_0/(1 - w_0)$ than the average gradient T'_+, and is equal to the latter in the absence of order, i.e., $w_0 = 1/2$. Expression (5), which connects the amplitude of the temperature fluctuations, is postulated in the semi-empirical turbulence theories that make use of the concept of the mixing length l_{mix}.[2] In our case $l_{\text{mix}} = x_0 w_0 / 2(1 - w_0)$.

The coefficient of strongly turbulent thermal conductivity is estimated by the expression $\chi_{10}^{(ST)} \approx x_0^2 \gamma_0/4$.[5] If the factor of T'_+ in (6) is treated as the coefficient of moderately turbulent thermal conductivity, then it is seen that it is $[\omega_0/(1-w_0)]^2$ times larger than $\chi_{10}^{(ST)}$ and coincides with the latter at $w_0 = 1/2$, i.e., in the absence of order.

We note that ordering of overlapping cells is similar to excitation of a quasi-mode,[7] the only difference being that the former is due to nonlinear interaction of the cells, and the latter is possible in the linear stage if the initial conditions are specially chosen.

An approach analogous to the proposed one is valid in the analysis of electrostatic convective cells[2] and of the overlap of split magnetic surfaces[8] that appear when drift instabilities develop. In the last case, the ordering of the magnetic islands leads not to stochastic diffusion[8,9] but to transverse convection of magnetic force lines and the associated anomalous electron thermal conductivity.

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[1)]Moderate turbulence is defined as a state in which the oscillations are strongly nonlinear but the motion correlation time greatly exceeds the characteristic time of the nonlinear interaction.

[2)]See[5,6] concerning the use of this approach to describe current-convective instability.

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