Unitary sum rules and collision times in strong overlap of resonance levels

V. L. Lyuboshitz

Joint Institute for Nuclear Research
(Submitted 23 April 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 1, 32–37 (5 July 1978)

The unitary sum rules obtained previously by the author [Sov. J. Nucl. Phys. **27**, No. 4 (1978); JINR preprint R4-10618, Dubna, 1977; Phys. Lett. **72B**, 41 (1977)] are used to investigate the time dependence of resonance processes at very high level density of the compound nucleus.

PACS numbers: 24.30.–v

In connection with the possibility of determining by experiment the lifetimes of compound nuclei with the aid of the "shadow effect" (see e.g.,[1] as well as[2]) it is of interest to estimate theoretically the durations of nuclear reactions in the case of strong overlap of the resonance levels. It was shown[3–6] that if the level density is very high, then the collision time becomes much larger than the reciprocal width $\hbar/\Gamma$ of the quasistationary states, and the time evolution of the resonant process has patently a non-exponential character. This effect is in final analysis the consequence of the S-matrix unitarity, which leads to destructive interference of the levels.

The author has derived previously[5,6] unitarity sum rules that connect the delay times of various resonance-reaction channels. It follows from these sum rules, in particular, that if all the S-matrix elements averaged over a sufficiently broad energy spectrum of the wave packet are close to zero, then in the case of n equivalent channels the average collision duration is

$$\bar{\tau} = \frac{\pi\hbar}{nD} >> \frac{\hbar}{\Gamma}, \tag{1}$$

where D is the distance between the neighboring levels. The "strong absorption" regime,[7] to which formula (1) is applicable, is realized in the limiting case $\Gamma \gg nD$ and corresponds to the *minimal* correlation between the decay amplitudes of the overlapping levels, which is needed to ensure under these conditions unitarity of the S matrix. If, on the contrary $\Gamma \ll nD$, then the indicated correlation between the resonance parameters can be neglected (the Bethe approximation, see[8,9]), and we obtain for the lifetime of the compound nucleus the "usual" result.

This paper deals with the temporal characteristics of nuclear reactions at arbitrary values of the parameter Γ/nD. We take as the basis the previously obtained relation[5,6]

$$-i\,\mathrm{Sp} < \frac{d\hat{S}(E)}{dE}\hat{S}^{+}(E) > = \frac{2\pi}{D}, \tag{2}$$

where $\hat{S}(E)$ is a unitary resonant S matrix corresponding to a definite value of the angular momentum, and the symbol $\langle\ \rangle$ denotes averaging over the energy spectrum with an energy spread $\Delta E \gg \Gamma$ or $\Delta E \gg D$. Since the S-matrix elements are bounded, we can neglect the quantities $\langle dS_{ij}(E)/dE\rangle$ and write accordingly

$$\mathrm{Im}\,\mathrm{Sp}\left\langle \frac{d\hat{T}(E)}{dE}\hat{T}^{+}(E)\right\rangle = \frac{2\pi}{D}, \qquad \hat{T}(E) = \hat{S}(E) - I. \tag{3}$$

This leads to the sum rule[5,6]

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\bar{\tau}_{ij} < |S_{ij}(E) - \delta_{ij}|^2 > = \frac{2\pi\hbar}{D}, \quad \sum_{i=1}^{n}\bar{\tau}_i (1 - \mathrm{Re} < S_{ii}(E) >) = \frac{\pi\hbar}{D}. \tag{4}$$

Here $\bar{\tau}_{ij}$ is the average delay time for the $i \to j$ transition, $\bar{\tau}_i$ is the average duration of the collision in the entrance channel[5,10]:

$$\bar{\tau}_{ij} = \frac{\mathrm{Im} < \frac{dT_{ij}(E)}{dE} T_{ij}^{*}(E) >}{< |T_{ij}(E)|^2 >}, \quad \bar{\tau}_i = \frac{1}{2}\,\frac{\sum_j \bar{\tau}_{ij} < |T_{ij}(E)|^2 >}{1 - \mathrm{Re} < S_{ii}(E) >}. \tag{5}$$

According to the known treatment of Friedman and Weisskopf,[11] the interaction of a particle with a nucleus evolves in time through an "instantaneous" stage and a stage of compound-nucleus formation. The effective cross sections, expressed in units of $(\pi/k^2)(2J+1)$, are equal to

$$\sigma_{ij}^{(\mathrm{inst})} = |< S_{ij}(E) > - \delta_{ij}|^2, \quad \sigma_{ij}^{(\mathrm{c.n.})} = < |S_{ij}(E)|^2 > - |< S_{ij}(E) >|^2. \tag{6}$$

We introduce now an average delay time $\tilde{\tau}_{ij}$ and an average collision duration $\tilde{\tau}_i$ for the compound-nucleus stage. Taking (6) into account,

$$\tilde{\tau}_{ij} = \bar{\tau}_{ij} \frac{\langle |S_{ij}(E) - \delta_{ij}|^2 \rangle}{\langle |S_{ij}(E)|^2 \rangle - |\langle S_{ij}(E) \rangle|^2}, \quad \tilde{\tau}_i = 2\bar{\tau}_i \frac{1 - \mathrm{Re}\langle S_{ii}(E) \rangle}{t_i}, \tag{7}$$

where $t_i = \Sigma_{j=1}^{n} \sigma_{ij}^{(\mathrm{c.n.})} = 1 - \Sigma_{j=1}^{n} |\langle S_{ij}(E) \rangle|^2$ is the transmission coefficient.[7,9] Then the second sum rule (4) is rewritten in the form

$$\sum_{j=1}^{n} \tilde{\tau}_i \, t_i = \frac{2\pi\hbar}{D}. \tag{8}$$

We continue the analysis within the framework of the model described in[5], where equivalence of the channels is assumed. We use the following relations of this model:

$$|\langle S_{ij}(E) \rangle| = \exp\left(-\pi\frac{\Gamma}{nD}\right), \quad t_i = 1 - \exp\left(-2\pi\frac{\Gamma}{nD}\right); \tag{9}$$

$$\langle \hat{S}(E)\hat{S}^{+}(E-\epsilon) \rangle_{ij} = \exp\left(i\frac{2\pi}{nD}\frac{\epsilon}{1 - i\epsilon/\Gamma}\right)\delta_{ij} \;^{1)}. \tag{9'}$$

Assuming that during the stage of the compound nucleus the delay times are the same for all the processes, we obtain on the basis of (8) and (9)

$$\tilde{\tau}_{ij} = \tilde{\tau}_i = \tilde{\tau} = \frac{2\pi\hbar}{nD}\left[1 - \exp\left(-2\pi\frac{\Gamma}{nD}\right)\right]^{-1}. \tag{10}$$

In the limiting cases $\Gamma \gg nD$ and $\Gamma \ll nD$, formula (10) yields

$$\tilde{\tau} = \begin{cases} \dfrac{2\pi\hbar}{nD}, & \Gamma \gg nD; \\ \hbar/\Gamma, & \Gamma \ll nD. \end{cases} \tag{11}$$

Inasmuch as at $\Gamma \gg nD$ we have $\langle S_{ij}(E) \rangle \approx 0$ (the analogy with diffraction by a black sphere), half of all the interaction acts pertain to the stage of the instantaneous elastic scattering, and in accord with (7) we have $\tilde{\tau} = 2\bar{\tau}$. This leads to the result (1). If $\Gamma \ll nD$ and there is no nonresonant background, then $t_i = 2\pi\Gamma/nD$, $\mathrm{Re}\langle S_{ii}(E) \rangle \approx 1 - (\pi\Gamma/nD)$, and $\tilde{\tau} = \bar{\tau} = \hbar/\Gamma$[5]; the probability of the instantaneous stage is then close to zero.

Consider the probability distribution of the delay time in the compound-nucleus stage. According to[4,5]

$$P_{ij}(\tau) = \frac{1}{2\pi\hbar}\int \tilde{\phi}_{ij}(\epsilon) e^{-i\epsilon\tau/\hbar} d\epsilon,$$

where $\tilde{\phi}_{ij}(\epsilon)$ are correlation functions that coincide with the coefficients of the correlation amplitudes in the theory of Ericson fluctuations[3]:

$$\tilde{\phi}_{ij}(\epsilon) = \frac{\langle S_{ij}(E) S^*_{ij}(E-\epsilon)\rangle - |\langle S_{ij}(E)\rangle|^2}{\langle |S_{ij}(E)|^2\rangle - |\langle S_{ij}(E)\rangle|^2}. \tag{12}$$

Assuming that all the processes have the same time variation during the compound-nucleus stage $[\tilde{\phi}_{ij}(\epsilon) \equiv \tilde{\phi}(\epsilon)]$ and using formula (9′), we obtain

$$\tilde{\phi}(\epsilon) = \left[\exp\left(i\frac{2\pi\epsilon}{nD(1-i\epsilon/\Gamma)}\right) - \exp\left(-\frac{2\pi\Gamma}{nD}\right)\right]\left[1 - \exp\left(-\frac{2\pi\Gamma}{nD}\right)\right]^{-1}. \tag{13}$$

It follows from (13) that the mean squared delay time during the stage of the compound nucleus is

$$\tilde{\tau^2} = -\hbar^2\frac{d^2\tilde{\phi}(\epsilon)}{d\epsilon^2}\Big|_{\epsilon=0} = \frac{\hbar^2}{\Gamma^2}\frac{x(x+2)}{1-e^{-x}}, \qquad x = \frac{2\pi\Gamma}{nD},$$

and the relative fluctuation is

$$\eta_\tau = \sqrt{\frac{\tilde{\tau^2} - (\tilde{\tau})^2}{(\tilde{\tau})^2}} = \left[\frac{2}{x}(1-e^{-x}) - e^{-x}\right]^{1/2}. \tag{14}$$

The function $\eta_\tau(x)$ decreases with increasing x. At $\Gamma \gg nD$ $(x \gg 1)$ we have $\eta_\tau = \sqrt{nD/\pi\Gamma} \ll 1$. In this limiting state the compound-nucleus stage is separated from the stage of i instantaneous elastic scattering by a time interval $\tilde{\tau} = 2\pi\hbar/nD$ and has a comparatively short time duration $\Delta\tau \sim \tilde{\tau}\sqrt{nD/\pi\Gamma} \ll \tilde{\tau}$. On the other hand if $\Gamma \ll nD$, i.e., $x \ll 1$, then we have from (13) that $\tilde{\phi}(\epsilon) = [1 - i(\epsilon/\Gamma)]^{-1}$, and the delay time has an exponential distribution; in this case $\eta_\tau = 1$.

It is known that the lower the rank of the matrix $\hat{T} = \hat{S} - I$ the stronger the correlation between the decay amplitudes of the different resonance levels.[9] It must be emphasized in this connection that the results (1), (10)–(11), and (13)–(14) correspond to a situation wherein the T-matrix has a *maximal rank* equal to the number n of the channels (all the eigenvalues of the T-matrix are different from zero, and those of the S matrix differ from unity). In our analysis, in the $\Gamma \gg nD$ limit, the compound-nucleus production cross section summed over the partial waves is equal to the geometric cross section. This picture agrees with experiment and is natural from the point of view of the universally accepted ideas concerning the structure of highly excited nuclei. Yet from the logical point of view other models are also possible and correspond to lower ranks of the T matrix. One such possibility was considered by Baz',[11] who expressed the unitary S matrix in the form

$$\hat{S}(E) = I + (e^{2i\delta(E)} - I)\hat{P}(E), \tag{15}$$

where $\delta(E)$ is the phase of the single-channel scattering, and P is a real projection matrix ($\widehat{P}^2 = \widehat{P}$, Sp$\widehat{P} = 1$). This parametrization correspond to the so-called "Newtonian" case of complete correlation between the decay amplitudes of all the resonances. The rank of the T matrix is then equal to unity. This regime was already discussed in the literature and was rejected because, contradicting the experimental data at a large number of channels, it leads to cross sections that are very small with the geometrical value πR^2.[8,9] It can be shown that if the T-matrix rank is $m(1 \leqslant m \leqslant n)$, then, provided that $\Gamma/mD \gg 1$, m eigenvalues of the matrix $\langle S(E)\rangle$ are equal to zero, and the remainder are equal to unity, so that the equality $\Sigma_{i=1}^{n}[1 - \mathrm{Re}\langle S_{ii}(E)\rangle] = m$ is satisfied. Taking this into account, the second sum rule in (4) yields

$$\bar{\tau}(max) \geqslant \frac{\pi\hbar}{mD} \gg \frac{\hbar}{\Gamma}, \quad \bar{\tau}(min) \leqslant \frac{\pi\hbar}{mD}.$$

Consequently, if all the times $\bar{\tau}_i$ are equal, they take on the value $\pi\hbar/mD$, which coincides with (1) at $m = n$ and with the result of[12] at $m = 1$. However, the realization of models corresponding to ranks $m < n$ seems to us very unlikely [the absorption cross section in such models is of the order of $\pi R^2(m/n) < \pi R^2$].

The author is deeply grateful to M.I. Podgoretskiĭ for a discussion and valuable advice.

[1)]Formulas (9) follow directly from the equality $|\det\langle\widehat{S}(E)\rangle| = \exp[-\pi(\Gamma/D)]$ obtained in[7] (in this case Γ has the meaning of the average total width); expression (9) is valid for a Poisson distribution of the resonances subject to the additional assumption that all the widths are equal.[2]

[1]S.A. Karamyan, Yu. V. Melikov, and A.F. Tulinov, Fiz. Elem. Chastits At. Yadra **4**, 456 (1973) [Sov. J. Part. Nucl. **4**, 196 (1973)].
[2]V.G. Baryshevskiĭ and V.I. Tkacheva, Dokl. Akad. Nauk BSSR **22**, 29 (1978).
[3].L. Lyuboshitz and M.I. Podgoretskiĭ, Yad. Fiz. **24**, 214 (1976) [Sov. J. Nucl. Phys. **24**, 110 (1976)].
[4].I. Kopylov, V.L. Lyuboshitz, and M.I. Podgoretskiĭ, JINR Commun. R2-9688, Dubna, 1976.
[5]V.L. Lyuboshitz, Yad. Fiz. **27**, 948 (1978) [Sov. J. Nucl. Phys. **27**, in press (1978)]; JINR R4-10618, Dubna, 1977.
[6]V.L. Lyuboshitz, Phys. Lett. B **72**, 41 (1977).
[7]M. Simonius, Phys. Lett. B **52**, 279 (1974).
[8]R.G. Thomas, Phys. Rev. **97**, 224 (1955).
[9]A. Lane and R. Thomas, *R*-matrix Theory of Nuclear Reactions, Rev. Mod. Phys. **30**, 257 (1958).
[10]T. Ohmura, Prog. Theor. Phys. **29**, 108 (1964).
[11]F.L. Friedman and V.P. Weiskopf, in: Niels Bohr and Development of Physics, London, 1955, p. 134.
[12]A.I. Baz', Pis'ma Zh. Eksp. Teor. Fiz. **27**, 142 (1978) [JETP Lett. **27**, 132 (1978)].