Polarized quarks and quantum chromodynamics

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We calculate the cross section for the scattering of ultrarelativistic polarized quarks in quantum chromodynamics in the Born approximation, which is valid at large momentum transfer because of asymptotic freedom. We discuss possible applications to elastic large-angle scattering of polarized protons, to production of particles with large transverse momenta, and to lepton-pair production processes in interaction of polarized hadrons.

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The significant role played by polarization effects in hadron collisions at high energy was demonstrated by experiments on elastic large-angle scattering of proton[1] and on inclusive production of λ particles in proton-proton and proton-nucleus reactions.[2] From the theoretical point of view, there is hope that polarization effects will not contain any new functions besides the customarily employed form factors, which alter so strongly the characteristics of processes with unpolarized particles (see, e.g.,[3]).

To understand the processes of scattering of polarized hadrons it is important to know the properties of the scattering of the valence quarks that make up the hadrons, a scattering that proceeds on account of gluon exchange. By virtue of asymptotic freedom, in quantum chromodynamics the process of scattering of two quanta with large momentum transfer is determined by the Born term[1)], which takes the form

$$M = \pi \alpha_s \left[\lambda^a_{\alpha\beta} \lambda^a_{\gamma\delta} \frac{(\bar{u}_3 \gamma_\mu u_1)(\bar{u}_4 \gamma_\mu u_2)}{(p_1 - p_3)^2} - \lambda^a_{\alpha\delta} \lambda^a_{\gamma\beta} \frac{(\bar{u}_4 \gamma_\mu u_1)(\bar{u}_3 \gamma_\mu u_2)}{(p_2 - p_3)^2} \right] \tag{1}$$

for the process of scattering of quarks with identical aromas $1(\alpha)+2(\gamma)\to 3(\beta)+4(\delta)$ with momenta p_1, p_2, p_3, and p_4 and with color indices α, β, γ, δ (λ^a are matrices of the $SU(3)$ group in color, u_i are the usual bispinor amplitudes, and $\alpha_s \equiv g^2$ is the square of

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the interaction constant). Expression (1) differs from the well known matrix element of the interaction of two electrons only in the presence of the color factors (and of course, in the substitution $\alpha_s \longleftrightarrow \alpha$). It is precisely they which determine the difference in the behavior of the cross sections of the two processes. Neglecting the quark masses in the ultrarelativistic limit, it is easy to calculate the cross section for the elastic scattering of two polarized quarks in the c.m.s.:

$$\frac{d\sigma}{dt} = \frac{2\pi\alpha_s^2}{s^2} N\Bigg[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + k\frac{2s^2}{ut} - (\vec{\zeta}_1\mathbf{n}_1)(\vec{\zeta}_2\mathbf{n}_1)$$
$$\times\left(\frac{s^2-u^2}{t^2} + \frac{s^2-t^2}{u^2} + k\frac{2s^2}{ut}\right) + 2k(\vec{\zeta}_{1\perp}\vec{\zeta}_{2\perp}) - k\frac{s^2}{ut}(\vec{\zeta}_{1\perp}\mathbf{n}_3)(\vec{\zeta}_{2\perp}\mathbf{n}_3)\Bigg], \tag{2}$$

where s, t, and u are the usual kinematic invariants, ζ_i is a unit vector along the polarization direction of the ith quark $\mathbf{n}_i=\mathbf{p}_i/|\mathbf{p}_i|$, the symbol $\perp$ labels the ζ_i vector component perpendicular to the quark collision axis, and summation is carried out over the polarizations of the quarks in the final state. The normalization factor N and the coefficient preceding the interference term k depend on the colors of the quarks and are listed in Table I.[2] $N=k=1$ for ee scattering ($\alpha_s \longleftrightarrow \alpha$). As seen from Table I

TABLE I. The symbol $\Sigma_{\beta\delta}$ denotes summation over the possible colors of the final quarks only while $\Sigma_{\alpha\beta\gamma\delta}$ denotes also averaging over the colors of the initial quarks.

	$\alpha=\gamma;\ \Sigma_{\beta\delta}$	$\alpha\neq\gamma,\ \Sigma_{\beta\delta}$	$\Sigma_{\alpha\beta\gamma\delta}$
N	1/9	5/18	2/9
k	1	− 3/5	− 1/3

and from formula (2), quarks of like color are scattered like electrons, apart from a coefficient, whereas scattering of quarks of different colors (as well as scattering averaged over all colors) is characterized by destructive interference, since the sign of the coefficient k is negative.

Reversal of the sign of k is particularly important in the case of scattering of transversely polarized quarks. For these quarks the effect is the opposite of that for electrons: for example, ultrarelativistic quarks polarized perpendicular to the scattering plane are scattered more strongly in a state with antiparallel spins than with parallel spins. The deviation from the unpolarized cross section increases with increasing scattering angle and is maximal at 90°.

We rewrite (2) in the form

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}(1 \pm C), \tag{3}$$

where the upper (lower) sign pertains to the case of parallel (antiparallel) spins.

We consider the most unfavorable case of scattering of quarks averaged over the initial colors and summed over the final colors, when the coefficient k has the smallest absolute value. It is easy to deduce from (2) at 90° in the c.m.s. that for helical quarks we have $C_{\parallel}(\pi/2) = -5/11$ (for helical electrons $C_{\parallel} = -7/9$), for quarks polarized perpendicular to the scattering plane we have $C_{\perp}(\pi/2) = -1/11$ (for electrons $C_{\perp} = 1/9$) and for quarks polarized in the scattering plane perpendicular to the collision axis $C_p(\pi/2) = 1/11$ (for electrons $C_p = -1/9$). Thus, for transversely polarized quarks the difference between the cross sections is about 20%, and in the case of helical quarks their ratio reaches 8/3.

What are the consequences of these results for processes at high energies?

First, let us discuss elastic scattering of polarized protons at large angles.[1] Quantitative conclusions can be obtained only by using a concrete proton-interaction model.[3] However, even qualitatively it can be seen that the main contribution should be made by scattering of quarks of like colors, inasmuch as in experiment the value of $C_{\perp}$ for protons is approximately 10^{-1} at $p_T^2 \lesssim 3.6$ GeV^2 and increases to 3×10^{-1} in the interval $3.6 \lesssim p_T^2 \lesssim 4.3$ GeV^2. It is possible also that the polarization properties of the proton are determined not by the valence quarks alone,[6] and in the construction of the model we must take into account the polarization of the gluons and of the quark-antiquark sea. We note that if the quarks have mass (estimated by some at about 300 MeV), the ultrarelativism condition may not be satisfied at $s = 25$ GeV^2 (the proton energy in the experiment of[1]).

An important fact is that a unique scaling holds for the quantities C. It is deduced from (3) and (2) that these quantities depend only on the ratio p_T^2/s. It follows therefore, in particular, that they are the same for all the quarks of the proton, regardless of the energy fraction x belonging to the given quark. This means that they do not depend on the forms of the distribution functions of the quarks inside the proton.

This, in particular, is important in the analysis of processes wherein particles are reproduced with large transverse momentum scattering on account of scattering of quarks by quarks[3] in the case of collisions of polarized protons. The jet azimuthal-asymmetry effects can serve in this case as a test on the hypothesis that they are of quark origin.

Polarization effects may turn out to be decisive also when it comes to explaining the mechanism whereby muon pairs are produced in proton-proton collisions. This can be easily understood if it is recognized that in the Drell-Yan mechanism (annihilation of a virtual quark and a virtual antiquark with conversion into a muon pair) color plays no role whatever (electromagnetic interactions), whereas the process of gluon-quark interaction, proposed in[7], does depend on color.

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[1)]We note that at large momentum transfers the exchange of many gluons leads to the appearance of a form factor in front of the Born amplitude, but does not change its spin structure[4,5] (meaning also no change in the conclusions concerning the polarization dependences).

[2)]Formula (2) is valid for scattering of quarks of like aroma. In the case of different aromas, there are no exchange or interference terms (i.e., $k=0$ and we must equate to zero the second and fifth terms in (2), which contain u^2 in the denominator).

[3)]An important factor here, of course is the antisymmetry of the proton wave function with respect to color, since this antisymmetry forbids single-gluon exchange between hadrons.

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