Oscillations of the electromagnetic field near a plasma focus and a Z pinch

S. K. Zhdanov and B. A. Trubnikov

I. V. Kurchatov Institute of Atomic Energy
(Submitted 15 February 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 2, 61–65 (20 July 1978)

It is shown that the presence of a peripheral plasma surrounding the pinch leads to localization, in the vicinity of the pinch, of electromagnetic fields capable of accelerating particles. In the case of weak wave damping this mechanism can apparently explain the recently observed peaks in the energy spectrum of accelerated deuterons (N. V. Filippov *et al.*, 8th Europ. Conf. on Controlled Fusion and Plasma Physics, Prague, 1977, Vol. **1**, 63).

PACS numbers: 52.55.Ez, 52.75.Di

1. A large number (~ 10) of peaks were observed in recent experiments[1] in the energy spectrum of deuterons accelerated to energies $\sim$1–4 MeV in a plasma focus (see Fig. 1). In our opinion, the observed structure of the spectrum can be attributed to excitation, near the pinch, of standing monochromatic oscillations of the accelerating electric field E_z under conditions when the frequency spectrum is discrete and only the fundamental harmonic of the oscillations is accelerated. It is shown below that these conditions arise if the pinch is surrounded by a peripheral plasma with density on the order of $\sim 10^{12}$ cm^{-3}, which, in analogy with the earth's ionosphere, reflects the low-frequency radio waves. We note that the total time of formation of the accelerated particles is estimated from the experimental data as the interval $\Delta t \sim 10$ nsec after the instant when the "singularity" occurs, and in the presence of about 10 peaks this should correspond to an oscillation period ~ 1 nsec, which is close to the Larmor period of deuterons in a field $B \sim 10^6$ G at the boundary of a pinch with a radius $a \sim 0.2$ cm at a current $I \sim 1$ MA.

2. We consider the simplest model of a cylindrical Z pinch with a current I, surrounded by a plasma of constant density n_0. In this plasma there can propagate an extraordinary wave with components $E_{rz}(r,t) \sim \exp(-i\omega t)$, and for E_z we have the wave equation

$$\frac{d}{r\,dr} r \frac{dE}{dr} + \frac{\omega^2}{c^2}\epsilon_{\rm eff} E = 0;\quad \epsilon_{\rm eff} = \frac{\xi^2 - \eta^2}{\xi};\quad \xi = 1 - \sum_{e,i} \frac{\omega_0^2}{\omega^2 - \omega_B^2};$$
$$\eta = -\sum_{e,i} \frac{\omega_0^2}{\omega^2 - \omega_B^2}\frac{\omega_B}{\omega} \tag{1}$$

where $\omega_{0\alpha}$ and $\omega_{B\alpha}$ are the plasma and cyclotron waves ($\alpha = e,i$). In the limit as $m_e \to 0$ we obtain

$$\xi = 1 + \frac{a x^2}{1 - x^2} ; \quad \eta = \frac{a x^3}{1 - x^2} ; \quad \epsilon_{\text{eff}} = 1 - \frac{a}{(1-a)^2} - \frac{a^2 x^2}{1 - a} - \frac{a/(1-a)^2}{(1-a)x^2 - 1} , \tag{2}$$

where $x=\omega/\omega_{Bi}=r(\omega Mc^2/2Ie)$ and $\alpha=(\omega_{0i}/\omega)^2$ is a parameter.

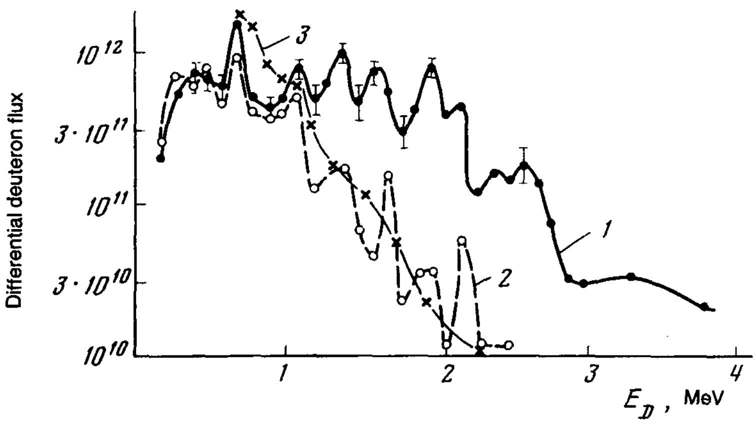

FIG. 1. Experimental energy spectrum of accelerated deuterons.[1] Curves 2 and 3, obtained respectively with the aid of the nuclear-emulsion method and by an activation procedure, pertain to one and the same discharge. The target activity for curve 1 is larger by one order of magnitude.

Assuming $\alpha \ll 1$, we use the approximation $\epsilon_{\text{eff}} = 1 - \alpha^2 x^2$. Equation (1) then reduces to the Schrödinger equation for a cylindrical oscillator

$$\frac{1}{x}\frac{d}{dx} x \frac{dE}{dx} + \beta^2 (1 - a^2 x^2) E = 0, \text{where} \beta = \frac{2eI}{Mc^3} << 1 \left(\beta \sim \frac{1}{30}\right), \tag{3}$$

whose quantum levels $\epsilon_{\text{osc}} = 2\hbar\omega_{\text{osc}}(n+\frac{1}{2})$ are equivalent in our case to the relation $\beta=(4n+2)\alpha$ that determines the frequency spectrum of the electromagnetic oscillations $\omega=\omega_n=\omega_{0i}\sqrt{(4n+2)/\beta}$. In a more rigorous treatment it is necessary to take into account the presence of ion hybrid resonance at the point $x_\infty = 1\sqrt{1-\alpha} \approx 1$, which we assume to coincide with the pinch boundary $r=a$, in view of the expected relation $\omega=\omega_{Bi}(r=a)$. Introducing $y=(1-\alpha)x^2, E=f(y)\exp(-\alpha\tilde{\beta}y/2)$, $\tilde{\beta}=\beta/(1-\alpha)^{3/2}$, we obtain from (1) and (2) the equation

$$y f''(y) + (1 - a\tilde{\beta}y) f'(y) + \frac{a\tilde{\beta}^2}{4}\left[a + \frac{1}{a} - 3 - \frac{2}{\tilde{\beta}} - \frac{1}{y-1}\right] f(y) = 0, \; y \gtrsim 1 \tag{4}$$

whose approximate solution at $\alpha \ll 1$ are the functions:

$$f(y)=\begin{cases} A_1\Psi(a,1,a\widetilde{\beta}y),\quad a=\frac{1}{2}-\frac{\widetilde{\beta}}{4}\left(a+\frac{1}{a}-3\right), \\ \text{at}\quad y-1\equiv\epsilon>>a \\ A_2\epsilon\exp\left(-\epsilon\frac{1-a\widetilde{\beta}+b}{2}\right)\Phi\left(1+\frac{a\widetilde{\beta}^2}{4b},\ 2,\epsilon b\right), \\ b=\sqrt{1+3a\widetilde{\beta}-\widetilde{\beta}^2},\quad \text{at}\quad \epsilon<<1 \end{cases} \tag{5}$$

which satisfy the condition $E_z\to 0$ as $y\to 1$. Here Φ and Ψ are confluent hypergeometric functions.[2] The joining together of the functions (5) in the common definition region $a\ll\epsilon\ll 1$ leads to the conditions

$$\Psi(a,1,a\widetilde{\beta})=0;\ A_2=-a\widetilde{\beta}\,\Psi(a,2,a\widetilde{\beta})A_1 \tag{6}$$

which determine the spectrum of the oscillations and the connection between the amplitudes.

It is easy to verify that, accurate to quantities of order $\delta=\ln^{-1}(2/\beta^2)\ll 1$, the spectrum at $\alpha\ll 1$ coincides with that obtained above. Thus, the presence of a peripheral plasma leads to formation, near the pinch, of a resonant cavity in which localized oscillations of the electromagnetic field can be excited.

3. We consider qualitatively three possible mechanisms whereby the described oscillations can build up. It is known[3] that in a number of experiments the plasma is heated by absorbing an external transverse wave in the region of the hybrid resonance. Obviously, however, an inverse effect is also possible—emission of an external wave produced by transformation from the "internal" waves, with a sufficiently high excitation level. Assume that the resonance point $x_\infty=(1-\alpha)^{-1/2}$ coincides with the surface of the pinch. Then the relation

$$E_r=-i\frac{\eta}{\xi}E_z=\frac{i a x^3}{(1-a)x^2-1}E_z \tag{7}$$

shows that in it $E_r\neq 0$ at $E_z=0$. Using formulas (5)–(7), we obtain, say for the fundamental mode, $\alpha=\beta/2(1+2\delta)$ and

$$E_z=-2i\,(E_r^0/\beta\delta)\,e^{-\beta^2 y/4}\,\Psi(-\delta,1,\beta^2 y/2),\ y-1>>a, \tag{8}$$

where $E_r^0=E_r|_{y=1}$ is the amplitude of the "internal" longitudinal wave. The maximum value of E_z is reached at the point $y_{\max}\approx\delta/\beta^2$, which corresponds to $r_{\max}/a\sim\sqrt{\delta}/\beta\sim 10$, with $|E_{z\max}/E_r^0|\sim 2/\beta\delta\gg 1$ and $|E_r(y_{\max})/E_{z\max}|\sim\sqrt{\delta}/2<1$, so that to accelerate the deuterons by a field $E_{z\max}$

within a time ~ 1 nsec to an energy ~ 4 MeV it suffices to have $E_r^0 \sim 10^2$ CGSE ($E_r^0/B_0 \sim 10^{-4} \ll 1$). Such fields E_r can be excited when a shock wave collapses and magnetosonic waves are excited in a pinch.

We note also the possibility of the buildup of a field E_z at the natural small oscillations of the pinch boundary.[4] Let $r_b(t) = a + s\cos\omega t$, $|s| \ll a$. Then, using the condition $E_z = -v_r B/c$ we obtain on the boundary, without allowance for the damping

$$E_z(r,t) = \frac{s\omega}{c} B_0 \frac{E_\omega(r)}{E_\omega(a)} \sin\omega t, \tag{9}$$

where $E_\omega(r)$ is the solution of the problem (1), (2) ($E_\omega \to 0$ as $r \to \infty$). If the frequency of the oscillations of the boundary ω is close to the natural frequency of the external cavity determined by condition (6), then we can expect resonant excitation of the field with an appreciable amplitude, since $E_\omega(a) \to 0$ as $\epsilon \to \omega_n$.

The most appreciable amplitude, however, is obtained if it is assumed that at the critical instant of the singularity $t = 0$ a sharp increase takes place in the turbulent resistivity of the pinch $\rho = (m\nu_{\text{eff}}/n_0 e^2)$; $\nu_{\text{eff}} = (e/m_e j_0)\Sigma_k \gamma_k (k/\omega_k) W_k$ apparently on account of the buildup of electrostatic electron cyclotron waves whose energy density $W = \Sigma_k W_k$, increases with a growth rate $\gamma \gtrsim \sqrt{|\omega_{Be}\omega_{Bi}|}$,[5] thus leading to practically instantaneous termination of the current I in the pinch. Then, using the approximation $\epsilon_{\text{eff}} = 1 - \alpha^2 x^2$, we obtain for the field $E_z = \Sigma_n E_n L_n(z) \times \exp(-z/2)\sin\omega_n t$, where $L_n(z)$ are Laguerre polynomials, $z = r^2\omega_{0i}^2/\beta c^2$ and $E_n = I\omega_n/c^2(n + 1/2)$, and at $n = 0$ we have in particular an amplitude $E_0 = \frac{1}{2}\beta B_0(r = a)$ if $\omega_{n=0} = \omega B_i(r = a)$. Such fields can also accelerate deuterons to observable energies of several MeV.

The authors thank M.A. Leontovich, N.V. Filippov, I.A. Belyaeva, T.I. Filippova, and Yu. S. Maksimov for useful discussions of the work.

[1]N.V. Filippov *et al.*, Eighth Europ. Conf. on Contr. Fusion and Plasma Phys. (Prague, 1977), Vol. 1, p. 63.
[2]A. Erdelyi, ed. Higher Transcendental Functions, McGraw, 1953 (Russ. transl., Nauka, 1965, Vol. 1, p. 237).
[3]V.E. Golant and A.D. Piliya, Usp. Fiz. Nauk **104**, 413 (1971) [Sov. Phys. Usp. **14**, 413 (1972)].
[4]B.A. Trubnikov, Fizika plazmy i problema UTR (Plasma Physics and Problems of Controlled Thermonuclear Reactions) AN SSSR **1**, 289 (1958).
[5]D.G. Lominadze, Tsiklotronnye volny v plazme (Cyclotron Waves in Plasma), Tbilisi, "Metsniereba," p. 122, 1975.