Nonlinear self-action of sound waves in an antiferromagnet with easy-plane anisotropy

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(Submitted 14 June 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 2, 93–97 (20 July 1978)

Results are reported of observation of nonlinear self-action of sound oscillations in single-crystal hematite. It is shown that the acoustic nonlinearity of the crystal, which is responsible for the effect, is due to magnetoelastic coupling.

PACS numbers: 75.80.+q, 75.50.Ee, 43.35.Rw

The acoustic properties of antiferromagnets with easy-plane anisotropy (EPAF), both linear and especially nonlinear, are determined to a considerable degree by the strong mutual influence of the elastic and magnetic subsystems of the crystal.[1-4]

In the present paper, using a hematite single crystal as an example, we examine the role of the magnetoelastic interaction in the formation of the cubic acoustic nonlinearity of EPAF.

The energy of long-wave acoustic oscillations of EPAF can be represented, accurate to terms of order higher than the fourth in the amplitudes, in the form

$$\mathcal{H} = \frac{1}{2}\sum_n \mu_n [A_n^2 + \Omega_n^2 A_n^2] - 2h_\perp(t)\sum_n G_n A_n + \sum_{mnq}\frac{1}{3!}\Phi_{mnq}A_m A_n A_q + \sum_{mnqq'}\frac{1}{4!}\Psi_{mnqq'}A_m A_n A_q A_{q'}, \quad (1)$$

$$\mu_n = \Omega_n^{-2}(\hat{C}_2 + \Delta\hat{C}_2)\int d\mathbf{r}\,\hat{U}_n^2, \quad (2)$$

$$\Phi_{mnq} = (\hat{C}_3 + \Delta\hat{C}_3)\int d\mathbf{r}\,\hat{U}_m\hat{U}_n\hat{U}_q, \quad (3)$$

$$\Psi_{mnqq'} = (\hat{C}_4 + \Delta\hat{C}_4)\int d\mathbf{r}\,\hat{U}_m\hat{U}_n\hat{U}_q\hat{U}_{q'}, \quad (4)$$

$$G_n = -\frac{(H + H_D)\hat{B}_2}{(\omega_{so}/\gamma)^2}\int d\mathbf{r}\,\hat{U}_n, \quad (5)$$

where Ω_n is the frequency of the normal acoustic (magnetoelastic mode) (we assume that $\Omega_n \ll \omega_{s0}$, ω_{s0} is the frequency of the antiferromagnetic resonance); $\hat{U}_n(\mathbf{r},t) = A_n(t)\hat{U}_n(\mathbf{r})$ is the strain in the normal mode; $\hat{C}_i$ are the elastic moduli of order i; $\hat{B}_1$ and $\hat{B}_2$ are the magnetoelastic-constant tensors; H and $h_\perp(t)$ are the constant and alternating magnetic fields, which are assumed to be oriented perpendicular to the basal plane: H_D is the Dzyaloshinskiĭ field. The magnetic corrections to the elastic moduli of second and third order are respectively[4]:

$$\Delta \hat{C}_2 = -\frac{H_E}{M_0}\left(\frac{2\hat{B}_2}{\omega_{s0}/\gamma}\right)^2,$$

$$\Delta \hat{C}_3 = -3\left(\frac{4H_E}{M_0}\right)^2 \frac{\hat{B}_1 \hat{B}_2^2}{(\omega_{s0}/\gamma)^4},$$

where H_E is the exchange field.

The cubic acoustic nonlinearity is due to the interaction with amplitude (4). The magnetic contribution to the fourth-order elastic moduli is determined by the relation

$$\Delta \hat{C}_4 = 12\left(\frac{H_E}{M_0}\right)^3 \frac{(2\hat{B}_2)^4}{(\omega_{s0}/\gamma)^6}\left(1 + \frac{1}{4}\frac{\gamma^2 H H_D}{\omega_{s0}^2}\right). \tag{6}$$

The cubic acoustic nonlinearity leads to self-action of the sound waves, which can manifest itself in the form of a nonlinear frequency shift of the sample magnetoelastic oscillations. In the case of resonant excitation of the crystal by a harmonic alternating field $h_\perp(t) = h_\perp \cos \omega t$, the amplitude-frequency characteristic of the forced acoustic oscillations is formally analogous to the corresponding hysteresis characteristic of a nonlinear oscillator:

$$2\frac{\Delta\omega}{\Omega_n} = -\frac{R_n}{\Omega_n^2}|b_n|^2 \pm \left[\left(\frac{\mu_n^{-1} G_n h_\perp}{\Omega_n^2 |b_n|}\right)^2 - Q_n^{-2}\right]^{1/2}, \tag{7}$$

where $2b_n$ is the amplitude of the oscillation with frequency ω, $\Delta\omega = \omega - \Omega_n$, and $R_n = \frac{1}{2}\mu_n^{-1}\Psi_{nnnn} + \frac{5}{6}(\mu_n^{-1}\Omega_n^{-1}\Phi_{nnn})^2$.

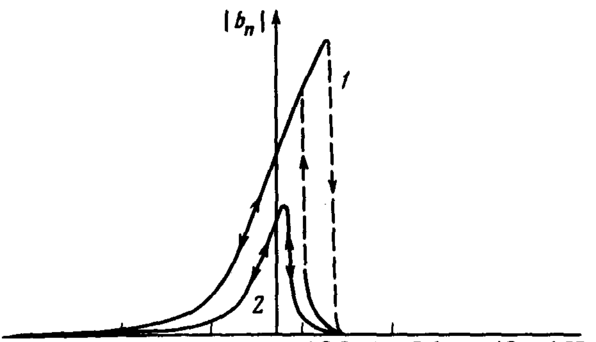

FIG. 1. Amplitude-frequency characteristics of acoustic resonance at $H = 2$ kOe: $h_{\perp 1}/h_{\perp 2} = 2$.

The nonlinear frequency shift was experimentally observed in a hematite single crystal excited at the transverse half-wave acoustic mode with a polarization perpendicular to the C_3 axis. The sample was a thin disk of 4.9 mm diameter and of thickness $l = 0.48$ mm, cut parallel to the basal plane. The sound oscillations were recorded by an induction method. The experimental amplitude-frequency characteristic shown in Fig. 1 agrees qualitatively with the theoretical characteristic (7). To assess the mechanism of the nonlinear frequency shift $\Delta\omega_N$ we measured its dependence on the intensity of the external magnetic field H ($\Delta\omega_N = \omega_N - \Omega_n$, where ω_N is the frequency corresponding to the maximum amplitude $2|b_n|$ at the given $h_\perp$). As applied to the considered oscillation mode, neglecting the intrinsic nonlinearity of the elastic subsystem, we can obtain from (7) the following relation for $\Delta\omega_N(H)$:

$$\frac{\Delta\omega_N}{\Omega_n} = \frac{9}{4}\left(\frac{C_{44}}{2B_{14}}\right)^2 \frac{\zeta_n^3}{1-\zeta_n}\left(1 + \frac{1}{4}\frac{\gamma^2 H H_D}{\omega_{s0}^2}\right)|b_n|^2, \qquad (8)$$

where $\zeta_n = (H_E/M_0)[2B_{14}/(\omega_{s0}/\gamma)]^2 + C_{44}^{-1}$ is the coupling coefficient that determines the linear renormalization of the frequency $\Omega_n^2 = C_{44}(1-\zeta_n)\pi^2/\rho l^2$ and is described for the investigated sample by the relation $\zeta_n = 0.42[H_{\mathrm{kOe}} + (H_{\mathrm{kOe}}^2/22) + 0.56]$ obtained from measuring the function $\Omega_n(H)$.

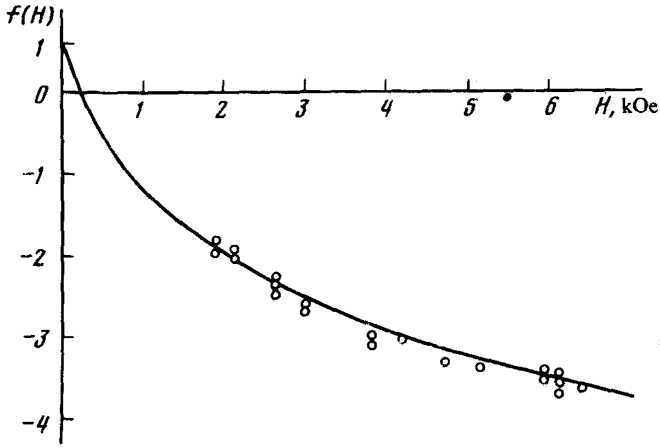

FIG. 2. Dependence of the nonlinear frequency shift on the magnetic field intensity: $f(H) = \lg\left[\frac{\Delta\omega_N}{\Omega_n}\Big/\left(\frac{C_{44}}{2B_{14}} b_n\right)^2\right]$.

Figure 2 shows the results of measurements of $\Delta\omega_N(H)$. The results of the calculation by formula (8) are shown by the continuous curve.

The strong field dependence of the nonlinear frequency shift, the value of which agrees with the expounded theory, shows that the nonlinear interaction of sound waves in hematite (at $H \lesssim 6$ kOe) is practically entirely due to magnetoelastic interaction.

We note that the nonlinear self-action in hematite manifests itself as the principal mechanism that limits the amplitudes of the parametric oscillations in experiments on parametric excitation of sound by longitudinal pumping.[5]

In conclusion, the authors thank N.N. Evtikhiev for constant interest in the work and for useful discussions of the results. The authors thank V.G. Kurilenko and A.A. Evdokimov for supplying the hematite single crystal, and V.F. Meshcheryakov for useful discussions.

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