Figure 2 shows the anisotropy b/a as a function of $E_{\gamma max}$. The anisotropy was obtained by least squares, by representing the angular distributions by the function $W(\theta) = a + b \sin^2\theta$ in nine intervals of the angle θ. The values of b/a were corrected for the content of even-even nuclei, using the anisotropy data from [7]. The data on the anisotropy indicate that it varies non-monotonically with changing sign in the region of 8.10 and 10.6 MeV. The negative anisotropy near 13.7 MeV corresponds apparently to the threshold of emission fission of U^{235}. The positive anisotropy at $E_{\gamma max}$ = 7 MeV was obtained from three measurements.

The presence of a peak in the cross-section curve at E_γ = 7 MeV and the presence, in part, of a dip in the 8 - 10 MeV region may be due to a difference in the sign of the anisotropy. The investigations are being continued.

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ELECTRIC BORN MODEL AND PION FORM FACTOR

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By choosing the electromagnetic pion form factor $F_\pi(k^2)$ in the form $(1 + 0.04k^2/m_\rho^2 - 0.108(k^2/m_\rho^2)^2)/(1 - k^2/m_\rho^2)$ for k^2 in the interval from 0.26 to 0.83 $(GeV/c)^2$ we obtain a satisfactory description of the experimental data on the electroproduction of π^+ mesons on hydrogen on the basis of the electric Born model.

The experiment performed at DESY [1] on the electroproduction of π^+ mesons on hydrogen is analyzed in the present article on the basis of the electric Born model (EBM) for the purpose of extracting information on the electromagnetic form factor $F_\pi(k^2)$ of the π meson.

At very small momentum transfers to the nucleon, the EBM calculations agree well with the results of experiments on high-energy $\pi^\pm$-meson photoproduction and high-energy ρ^0-meson production in the reaction $\pi^- + p \to \rho^0 + n$ [1]. In the latter reaction, in the spirit of the known ρ^0-γ analogy, the ρ^0 meson can be regarded as a virtual isovector photon γ_V^* of mass m_ρ. We propose to generalize the EBM to include electroproduction of charged pions, namely, for concreteness, to include the reaction $e^- + p \to e^- + \pi^+ + n$ at high energies of the final π^+n system and very low momentum transfers to the nucleon. In electroproduction in the one-photon approximation (OPA), the 4-momentum k of the virtual photon is space-like ($k^2 \leq 0$ in the chosen metric $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$), and the photon γ^* itself is assumed to be isoverctor (concerning the smallness of the contribution of the isoscalar photon component in the related photoproduction reaction see, e.g., Richter's paper [2]).

The differential cross section of pion electroproduction in the OPA is given by [1]

$$\frac{d^3\sigma}{dW^2 dt\, dk^2} = \frac{\alpha}{8\pi}\,\frac{1}{E_1^2 M^2(-k^2)}\,\frac{W^2 - M^2}{1-\epsilon}\left[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}\right], \qquad (1)$$

where ε is the polarization parameter of the exchanged photon, defined by

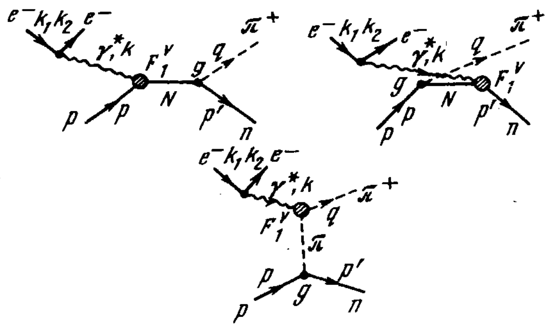

Fig. 1. Born diagrams for the reaction $e^- + p \to e^- + \pi^+ + n$.

$$\epsilon = \frac{1}{1 + 2\left[1 + \frac{(E_1 - E_2)^2}{(-k^2)} \operatorname{tg}^2 \frac{\theta}{2}\right]} \tag{2}$$

In (1) $d\sigma_T/dt$ and $d\sigma_L/dt$ are the differential cross sections of the reaction $\gamma^* + p \to \pi^+ + n$ for transversely and longitudinally polarized virtual photons; W is the total energy of the final $\pi^+ n$ system, t is the square of the momentum transferred to the nucleon, and M is the nucleon mass; $\alpha = e^2/4\pi = 1/137$ is the fine-structure constant. In (1) and (2), E_1 and E_2 are the energies of the incident and scattered electrons, respectively, and θ is the e^- scattering angle in the l.s. of the reaction $e^- + p \to e^- + \pi^+ + n$.

The EBM for the reaction $e^- + p \to e^- + \pi^+ + n$ in the OPA is determined by the Feynman diagrams of Fig. 1 [3, 4]. In the $\pi\pi\gamma_V^*$ and $\bar{N}N\gamma_V^*$ vertices, use is made of the electromagnetic form factor of the pion $F_\pi(k^2)$ and the electromagnetic Dirac isovector form factor of the nucleon $F_1^V(k^2)$, respectively (the contribution of the Pauli form factor is negligibly small at small momentum transfers to the nucleons). k_1 and k_2 denote the 4-momenta of the incident and scattered electrons; $k = k_1 - k_2$ is the 4-momentum of the virtual isovector photon γ_V^*; p, p', and q are the 4-momenta of the proton p, the neutron n, and of the π^+meson, respectively. The Mandelstam variables s and t are defined in the usual manner: $s = (p' + q)^2 = (p + k)^2$ and $t = (p - p')^2 = (k - q)^2$. $s = W^2$ in the c.m.s. of the reaction $\gamma^* + p \to \pi^+ + n$.

The diagrams of Fig. 1 yield the following summary contribution to the matrix element of the hadron current:

$$\langle \pi^+, n | J_\mu^h |_p \rangle = i\sqrt{2} g e F_\pi(k^2) \bar{U}_n(p')\gamma_5 \left[\frac{2q_\mu}{t - \mu^2} + \frac{p_\mu}{W^2 - M^2}\kappa + \right.$$
$$\left. + \frac{p_\mu}{W^2 - M^2 + t - \mu^2 - k^2}\kappa + \frac{(\gamma k)\gamma_\mu}{2(W^2 - M^2)}\kappa - \frac{\gamma_\mu(\gamma k)}{2[W^2 - M^2 + t - \mu^2 - k^2]}\kappa \right] u_p(p). \tag{3}$$

In (3), $\bar{u}_n(p')$ and $u_p(p)$ are the Dirac spinors of the nucleons, $\kappa = \kappa(k^2) = F_1^V(k^2)/F_\pi(k^2)$ with normalization $\kappa(0) = 1$, μ is the pion mass, and g is the NN_π coupling constant, chosen to equal $g^2/4\pi = 14.7$ in the calculations.

The hadron-current matrix element defined by (3) makes the following contributions to the differential cross sections in (1), under the condition $W^2 - M^2 \gg |t - k^2 - \mu^2|$:

$$\frac{d\sigma_T}{dt} = \frac{2\pi\alpha}{(W^2 - M^2)^2}\left(\frac{g^2}{4\pi}\right)\frac{F_\pi(k^2)^2}{(t - \mu^2)^2}\left\{t^2 + [t(1 - \kappa) + \mu^2\kappa]^2\right\} \tag{4}$$

and

$$\frac{d\sigma_L}{dt} = -\frac{2\pi\alpha}{(W^2 - M^2)^2}\left(\frac{g^2}{4\pi}\right)\frac{F_\pi(k^2)^2}{k^2(t - \mu^2)^2}[k^2 + (1 - \kappa)(t - \mu^2)]^2. \tag{5}$$

As shown by a comparison of experiments on the photoproduction of $\pi^\pm$ mesons and the production of ρ^0 mesons in the reaction $\pi^- + p \to \rho^0 + n$ with the predictions of the EBM, we can expect expressions (4) and (5) to be valid at $|t| \leq 2\mu^2 \approx 0.04$ (GeV/c)^2 and $W^2 - M^2 \geq 3.5$ (GeV/c)^2. (We note that any difference we obtain between the EBM for $d\sigma_T/dt$ and experiments on $\pi^\pm$-meson photoproduction ($d\sigma_L/dt \equiv 0$ in this case) will be ascribed to a more complicated dependence on W than $(W^2 - M^2)^{-2}$ in (4), and we shall rais or lower the values of $d\sigma_T/dt$ and $d\sigma_L/dt$ for all k^2 by the same amount as used at $k^2 = 0$ for the normalization $F_\pi(0) = 1$.) In addition, one can easily note a strong dependence of $d\sigma_L/dt$ on $F_\pi(k^2)$ and a weak dependence on $F^V(k^2)$, owing to the presence of the small factor $(t-\mu^2)$ in front of κ. Thus, with 2 - 5% accuracy, we can put $\kappa(k^2) = 1$ in the combination of the cross sections $d\sigma_T/dt$ and $d\sigma_L/dt$ in (1), which is then given by

$$\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} = \frac{2\pi\alpha}{(W^2 - M^2)^2}\left(\frac{g^2}{4\pi}\right)\frac{F_\pi(k^2)^2}{(t-\mu^2)^2}[t^2 + \mu^4 + \epsilon t k^2] \qquad (6)$$

Comparison of (6) with the experimental data makes it possible to study directly the pion electromagnetic form factor $F_\pi(k^2)$. The cited DESY experiment was performed at W = 2.2 GeV at t = -0.037 $(GeV/c)^2$, and the combination $d\sigma_T/dt + \epsilon d\sigma_L/dt$ was measured at an average value of ϵ equal to 0.75, and at -k equal to 0.18, 0.26, 0.34, 0.48, 0.63, 0.68, and 0.83 $(GeV/c)^2$. comparing (6) with the indicated experiment for all k^2 except $k^2 = -0.18$ $(GeV/c)^2$, we have fitted the expression for $(1 - k^2/m_\rho^2)F_\pi(k^2)$ as follows:

$$\left(1 - \frac{k^2}{m_\rho^2}\right)F_\pi(k^2) = 1 + c_1\frac{k^2}{m_\rho^2} + c_2\left(\frac{k^2}{m_\rho^2}\right)^2. \qquad (7)$$

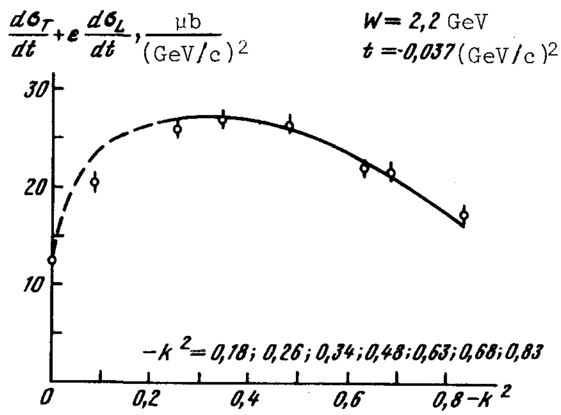

Fig. 2. Solid curve - values of $d\sigma_T/dt$ + $[\epsilon(d\sigma_L/dt)]$ obtained from expression (6), taken from (7) with c_1 = 0.04 and c_2 = -0.108 for k^2 in the interval 0.26 ≤ $-k^2$ ≤ 0.83 $(GeV/c)^2$. Dashed curve - continuation of (7) with the same c_1 and c_2 to k^2 = 0.

The least-squares method yields c_1 = 0.04 and c_2 = -0.108. χ^2 is then equal to 0.982, corresponding to a 90% confidence level. The solid line in Fig. 2 corresponds to the values $d\sigma_T/dt + \epsilon d\sigma_L/dt$ from (6), taken with (7) with c_1 = 0.04 and c_2 = -0.108. If the value of $F_\pi(k^2)$ at $k^2 = -0.18$ $(GeV/c)^2$ is included in the analysis, it becomes impossible to obtain a fit with a reasonable confidence level, since the value $F_\pi(k^2 = -0.18) = 0.67$ lies much lower than the values of F_π for neighboring k^2.

The causes of the small F_π at $k^2 = -0.18$ (Ge/c) may be: a) systematic errors in the experiment; b) the presence of additional contributions to the amplitudes of the reaction $\gamma_V^* + p \to \pi^+ + n$ at small k^2; c) anomalies in the behavior of the form factors at small k^2, predicted by a number of workers (see, e.g., [5]). Experiments are therefore necessary with small values of k^2 close to -0.18 $(GeV/c)^2$, other conditions being the same. Ascribing this difficulty to the systematic errors of the experiment, we can assume that (7) with the indicated c_1 and c_2 is valid for small k^2 down to $k^2 = 0$ (the dashed line in Fig. 2 corresponds to a continuation of (7) to $k^2 = 0$) and estimate the pion radius, defined as $r_\pi = \sqrt{6F_\pi'(0)}$, at 0.65 F.

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