DAMPING OF TRANSVERSE WAVES IN ROTATING HELIUM

V. K. Tkachenko
Institute of Solid State Physics, USSR Academy of Sciences
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The damping of transverse waves in rotating helium is calculated with allowance of the joint oscillations of the superfluid and normal components. It is found that at an oscillation period ∿1 min and an angular velocity of several radians the damping is weak enough and the transverse waves can be experimentally observed.

It is known that quantized vortices are produced in rotating superfluid helium. The circulation of the velocity around one vortex is equal to h/m, where h is Planck's constant and m is the mass of the helium atom. Calculations show that these vortices form a triangular lattice [1] possessing elasticity [2] with a certain shear modulus G. This causes transverse waves to propagate in the rotating helium, with a velocity s satisfying the equation $\rho_s s^2 = G$ if the interaction with the normal component is neglected. Here ρ_s is the density of the superfluid component. The velocity s depends on the angular velocity Ω of the rotation in accord with the formula [1] $s = (1/2)\sqrt{\hbar\Omega/m}$.

Let us examine the damping of these waves as a result of the interaction between the vortices and the normal component of the helium. Stauffer [3] has already calculated this damping, but under the assumption that the normal component is at rest. This led him to the conclusion that the damping of the transverse waves is very strong at T > 1°K, owing to the mutual friction of the normal and superfluid components. Actually, however, the normal component of the helium also takes part in the oscillations, and allowance for this fact leads to a much smaller damping, which makes the transverse waves experimentally observable. The damping is determined here mainly not by the mutual friction force but by the viscosity of the normal component.

The entire analysis is performed in the rotating reference frame. Let $\zeta = \zeta(x, t)$ and $\xi = \xi(x, t)$ be the displacements of the superfluid and normal components in the oscillations, respectively. The interaction force between the normal and superfluid components, due to the vortices, is then [4] $F[(\partial\xi/dt) - (\partial\zeta/dt)]$, where $F = B(\rho_s\rho_n/\Omega)$. Here B is the Hall-Vinen coefficient, ρ_s is the density of the normal component, and $\rho = \rho_s + \rho_n$. The wave equation for the lattice and the Navier-Stokes equation for the normal component assume, when this interaction is taken into account, the form

$$\rho_s \frac{\partial^2 \zeta}{\partial t^2} = G\frac{\partial^2\zeta}{\partial x^2} + F\left(\frac{\partial \xi}{\partial t} - \frac{\partial\zeta}{\partial t}\right),$$

$$\rho_n \frac{\partial^2 \xi}{\partial t^2} = \eta\frac{\partial^3\xi}{\partial t\partial x^2} - F\left(\frac{\partial \xi}{dt} - \frac{\partial\zeta}{dt}\right).$$

Here η is the viscosity of the normal component.

Assuming in the solution of the linear system, as usual, $\zeta = \mathrm{Re}\,\zeta_0\exp[i(kx - \omega t)]$ and $\xi = \mathrm{Re}\,\xi_0\exp[i(kx - \omega t)]$, we obtain the system

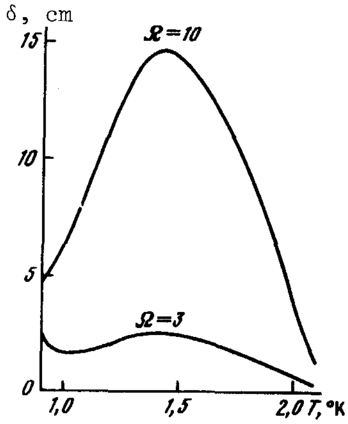
$$-\rho_s\omega^2\zeta_0 = -Gk^2\zeta_0 - iF\omega(\xi_0 - \zeta_0),$$

$$-\rho_n\omega^2\xi_0 = i\eta\omega k^2\xi_0 + iF\omega(\xi_0 - \zeta_0).$$

This yields an algebraic equation for the wave vector $k = k_1 + ik_2$:

$$\eta Gk^4 - (iG\rho_n\omega + \eta\omega^2\rho_s + iF\eta\omega - FG)k^2 + i\rho_s\rho_n\omega^3 - F\omega^2\rho = 0.$$

The figure shows the dependence of the depth of penetration of the wave $\delta = 1/k_2$ on the temperature T at ω = 0.1 rad/sec (oscillation period ∿1 min) for Ω = 3 rad/sec and Ω = 10 rad/sec. The wavelength at T < 2°K agrees well (within several per cent) with the formula $\lambda = (2\pi s/\omega)\sqrt{\rho_s/\rho}$ corresponding to complete dragging of the normal component. At sufficiently low temperatures, the depth of penetration increases, since the influence of the normal component on the oscillations becomes negligible. We note that, according to Reatto [5], the oscillations become damped, albeit very weakly, even at absolute zero temperature, owing to the finite propagation velocity of the interaction in the helium.



Penetration depth of transverse wave at ω = 0.1 rad

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EXCITATION OF NUCLEAR LEVELS BY ELECTRONS AND HADRONS AT HIGH ENERGY

L. A. Kondratyuk and Yu. A. Simonov
Institute of Theoretical and Experimental Physics

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Many experimental results on the cross sections for the excitation of nuclear levels by high-energy protons have been reported recently [1, 2]. The present paper aims to show that at high energy there exists a unique connection between the excitation cross sections of a definite series of nuclear transitions in electron and hadron beams. This series includes transitions that can be realized by exchange of states with natural parity in the t-channel, i.e., transitions of the type $0^+ \to 0^+, 1^-, 2^+, 3^-$, etc. Transitions of this type can be excited upon exchange of the vacuum quantum numbers in the t-channel, so that their excitation cross sections do not decrease with increasing energy [3].

To obtain the sought connection, we consider the scattering of a hadron by a nucleus within the framework of the multichannel optical model

$$(\nabla^2 + k_\alpha^2)\Psi_\alpha(r) = \sum_{\alpha'} V_{\alpha\alpha'}(r)\,\Psi_{\alpha'}(r), \qquad (1)$$

where Ψ_α is the wave function of a system consisting of a hadron and a target nucleus in the state α. A similar model was used to calculate the excitation cross sections of hadrons interacting with nuclei. In our case, the optical potential is given by

$$U_{\alpha\alpha'}(r) = -4\pi f(0) A \rho_{\alpha\alpha'}(r), \qquad (2)$$

where f(0) is the amplitude of the elastic hadron-nucleon scattering, and $\rho_{\alpha\alpha'}(\vec{r})$ is the single-particle density matrix, defined in terms of the form factor with the aid of the Fourier transformation

$$\rho_{\alpha\alpha'}(r) = \int e^{-iqr} S_{\alpha\alpha'}(q)\frac{d^3q}{(2\pi)^3}, \qquad (3)$$

$$S_{\alpha\alpha'}(q) = \frac{1}{A}\int\left(\sum_{i=1}^{A} e^{iqr_i}\right)\Psi_{\alpha'}^*(r_1, \ldots, r_A)\times \Psi_\alpha(r_1, \ldots, r_A)\,d^3r_1 \ldots d^3r_A. \qquad (4)$$

When considering any transition $i \to f$ we shall, by virtue of the smallness of the off-diagonal elements of the matrix ρ in comparison with the diagonal ones, take the potential V_{if} into account only in first order perturbation theory, while the potentials V_{ii} and V_{ff} will be taken