shows the distribution with respect to E_{exc} obtained experimentally for the reaction Li^6(p, 2p) at 155 MeV [2]. It is important that the distribution with respect to the excitation energy has a maximum in the region of the ground state of He^5.

If the data on the (p, 2p) reaction are reduced with allowance for the diagram 1a and a 'background' in the form of the diagram 1b, we obtain a good description of the experimental data (see Fig. 3; $\chi^2 = 23.5$ and 15.1 on Figs. 3a and 3b, respectively; we point out for comparison that $\chi^2 = 88$ and 212, respectively, for the distributions of [1]).

From the data on (p, 2p) at 155 MeV we get for the reduced proton width $\theta_p^2 = 0.39 \pm 0.06$, and at 185 MeV $\theta_p^2 = 0.27 \pm 0.06$ (at a channel radius 4 F). When account is taken of the diagram 1b, the reduction of the data on the reaction $Li^6(\pi^-, \pi^- p)$ yields $\theta_p^2 = 0.5 \pm 0.2$. The indicated values of θ_p^2 thus agree within the limits of errors. The contradiction between the values of θ_s^2, which were found in [3] to be too large in comparison with the reduced width of the s transition to the excited level of He^5 is likewise eliminated, since it is now clear that θ_s^2 takes effectively into account some of the transitions to the state He^4 + n via a mechanism corresponding to the diagram of Fig. 1b. The effective number of deuterons in Li^6, obtained from a description of the (p, 2p) data with the aid of the diagrams of Fig. 1, is found to be of the order of 0.4, which agrees with the known data on the (p, pd) reactions (4, 6].

These facts, as well as the significantly improved description of the experimental data by taking the diagram 1b into account, enable us to hope that the proposed model corresponds to the real situation. It should then affect also the characteristics of other reactions, for example Li^6(e, ep).

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SELF-CONSISTENCY CONDITIONS IN SYSTEMS WITH BROKEN SYMMETRY

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We derive the self-consistency conditions that must be satisfied by the mass operator, the density matrix, and the two-particle interaction in systems with broken symmetry.

We wish to call attention in this article to the fact that for systems with broken symmetry, which have been diligently studied of late[1], there exist self-consistency conditions that interrelate definite components of the mass operator Σ, of the density matrix ρ, and the two-particle interaction. They play an important role in the determination of the critical points and of the characteristics of the collective-excitation spectrum. We shall derive these conditions by using a generalized Ward identity. Using for the ψ operators the transformation $\psi(x) \to \exp[if(t)Q(\vec{x})]\psi(x)$, where $Q(\vec{x})$ is a certain time-independent Hermitian operator and f(t) is an arbitrary real function of the time, and using standard methods (see, e.g., [2]), we get[1)]

$$\omega\mathcal{T}(x, p, \epsilon, \omega; [\mathring{j}_Q]) + \mathcal{T}(x, p, \epsilon, \omega; [D_Q]) = G^{-1}\left(x, p, \epsilon + \frac{\omega}{2}\right)Q(x) - Q(x)G^{-1}\left(x, p, \epsilon - \frac{\omega}{2}\right). \quad (1)$$

[1)] We have changed over here to a mixed representation. In what follows, we shall use frequently a symbolic notation and omit the quantum numbers as well as the variables over which the summation and integration are carried out.

Here $G^{-1} = \varepsilon - \varepsilon_p^0 - \Sigma$ is the reciprocal Green's function, while $\mathcal{T}[j_Q^\circ]$ and $\mathcal{T}[D_Q]$ are vertices in which the role of the external field is played by the fourth component of the "Q-current" j_Q^ν and its "divergence" $iD_Q = \partial_\nu j_Q^\nu$.

We are interested in those cases when the right-hand side of (1) does not vanish identically when $\omega \to 0$. This can occur when: 1) the "Q-current" is not conserved (such a situation is considered in [2]) and 2) the "Q-current" is conserved but the symmetry of the considered state is broken, i.e., the mass operator is characterized by a nonzero macroscopic parameter that is not invariant relative to the given symmetry transformation, whereas the energy of the state is not altered by such a transformation. Such macroscopic parameters are the moment of inertia of the deformed drop (Σ does not commute with the angular-momentum operator $\vec{L}$), the isospin of the nucleus at $N \neq Z$ (Σ does not commute with the isospin operator $\vec{\tau}$), the coordinate of the center of mass (CM) of the final system, etc. In view of the indicated degeneracy, the standard methods of quantum theory cannot be applied directly, and it is necessary to lift the degeneracy somehow, for example by applying to the system an external field that does not commute with the given symmetry transformation [3]. If the state under consideration is stable, then a very weak field $V(\vec{x})$ suffices to lift the degeneracy and to "freeze" the degree of freedom corresponding to this parameter (if the changes of the external conditions bring the system close to the critical point, where it ceases to be stable, then the field necessary to "maintain" a certain fixed value of the macroscopic parameter increases sharply). Thus, to exclude the motion of the CM of a spherical liquid drop along the x axis, it suffices to place it in a square well $V(x)$ of depth V_0 such as to make the amplitude of the zero-point CM oscillations small compared with the distance between particles; it is only under this condition that it makes sense to measure the density. A simple estimate shows that the depth of the well should satisfy in this case the condition $V_0 \gg \varepsilon_0 A^{-4/3}$, where ε_0 is the characteristic energy, and A is the number of particles in the system. When the drop turns into vapor, the external field required to "secure" the CM is already $V_0 \sim \varepsilon_0$.

Application of an external field $V(\vec{x})$ gives rise to a "Q-current" divergence $D_Q \sim [Q, V]$. Letting the frequency ω tend to zero, we obtain from (1)

$$\mathcal{T}(x, p, \epsilon; [D_Q]) = G^{-1}(x, p, \epsilon)\, Q(x) - Q(x) G^{-1}(x, p, \epsilon) \equiv [G^{-1} \cdot Q]$$

It follows therefore that $\mathcal{T}(\omega = 0; [D_Q])$ does not depend on V_0. This means that the vertex $\mathcal{T}(\omega; [D_Q])$ has a pole near zero, i.e., $\mathcal{T}(\omega; [D_Q]) = -\omega_0[G^{-1}, Q]/(\omega - \omega_0)$ (the position of the pole ω_0 is proportional to the field V_0). As seen from (1), the poles $\mathcal{T}[j_Q^0]$ and $\mathcal{T}[D_Q]$ coincide, since there are no singularities at all in the right-hand side when ω is small. Therefore as $\omega \to \omega_0$ we have

$$\mathcal{T}[j_Q^\circ] = \frac{[G^{-1}, Q]}{\omega - \omega_0}. \tag{2}$$

The vertex $\mathcal{T}[j_Q^0]$ at the pole $\omega = \omega_0$ satisfies the standard homogeneous equation

$$\mathcal{T} = UG\mathcal{T}G, \tag{3}$$

where U is an irreducible block in the particle-hole channel. It follows from the foregoing that we are able to choose the external field V_0 small enough to make the frequency ω_0 small in comparison with the frequencies ω_1 of the single-particle transitions in the right-hand side of (3). Neglecting terms of order ω_0/ω_1, we obtain from (2) and (3) the self-consistency condition

$$\Sigma(x, p, \epsilon) Q(x) - Q(x) \Sigma(x, p, \epsilon) = \\ = \int \frac{dx' dp' d\epsilon'}{(2\pi)^4 i} U(x, p, \epsilon; x', p', \epsilon')[G(x', p', \epsilon') Q(x') - Q(x') G(x', p', \epsilon')]. \tag{4}$$

We now consider concrete examples. For systems with broken symmetry with respect to shear (liquid drops, crystals), the condition (4) takes the form

$$\frac{\partial \Sigma(x, p, \epsilon)}{\partial x} = \int \frac{dx' dp' d\epsilon'}{(2\pi)^4 i} U(x, p, \epsilon; x', p', \epsilon') \frac{\partial G(x', p', \epsilon')}{\partial x'}. \tag{5}$$

This relation can be normalized by the usual procedure of the theory of Fermi liquids, introducing the local amplitude Γ^ω of the quasiparticle interaction on the Fermi surface:

$$\frac{\partial \Sigma}{\partial x} = \Gamma^{\omega} A \frac{\partial G^{-1}}{\partial x} \quad . \tag{6}$$

Here A is the integral of the pole parts G^q of the two Green's functions with respect to ε; $G^q(\vec{x}, \vec{p}, \varepsilon) = a(\vec{x})[\varepsilon - \mathcal{H}^q(\vec{x}, \vec{p})]^{-1}$, where $\mathcal{H}^q = \vec{p}^2/2m^* + U$. We emphasize that (6) contains the mass operator Σ, and not the Hamiltonian of the quasiparticles $\mathcal{H}^q$ (they coincide only when $m^* = m$ and $a = 1$, where m^* is the effective mass and $\underline{a}$ is a renormalization factor). A decisive role is played in the nucleus by the zeroth harmonic of Γ^{ω}, i.e., $\Gamma^{\omega}(\vec{x}, \vec{x}') = \Gamma^{\omega}(x)\delta(\vec{x} - \vec{x}')$. In the particular case when $m^* = m$ and $a = \text{const}$, we obtain from (6) for spherical nuclei $\partial U(x)/\partial x = F(x)[\partial \rho(x)/\partial x]$, where $F(x) = a^2\Gamma^{\omega}(x)$ and $\rho(x)$ is the quasiparticle density. A similar relation in somewhat different form was obtained in [4].

The situation is somewhat different in systems with pairing, Owing to the appearance of a Bose condensate of the bound paires, Eq. (3) is no longer valid and a system of more complicated equations is necessary. It is nevertheless easy to show that in this case, too, the self-consistency condition for a system with violated translational invariance has a form similar to (5):

$$\frac{\partial \Sigma}{\partial x} = U \frac{\partial G_s}{\partial x} \tag{7}$$

but here the Green's function G_s takes pairing into account, and Σ does not include the transformation of a particle into a hole and a condensate pair. (Inasmuch as the gap Δ is determined from the homogeneous equation, no new conditions are obtained for it.)

For systems with broken rotational symmetry, the self-consistency equation (4) takes the form (in the coordinate representation):

$$\left(x \times \frac{\partial}{\partial x} + y \times \frac{\partial}{\partial y}\right)\Sigma(x, y, \epsilon) =$$

$$= \int \frac{dx'dy'd\epsilon'}{2\pi i} U(x, y, \epsilon; x', y', \epsilon')\left[x' \times \frac{\partial}{\partial x'} + y' \times \frac{\partial}{\partial y'}\right]G(x', y', \epsilon'). \tag{8}$$

In the same approximation as above, retaining only the zeroth harmonic, we obtain for an axially-symmetrical deformed drop the simple relation $\partial U(r,\theta)/\partial\theta = F(r, \theta)[\partial\rho(r, \theta)/\partial\theta]$.

We emphasize that for anisotropic media and deformed systems the conditions (8) and (5) should be satisfied simultaneously.

Equation (4) can be generalized to include the case of multicomponent systems (ionic crystals, atoms, etc.); then U, G, and Σ become matrices.

In some cases, spontaneous symmetry breaking can appear simultaneously relative to discrete and continuous transformation groups. Thus, Migdal [5] has shown that particle-hole pairing (pion condensate) occurs in nuclear matter under ordinary circumstances. As a result, a term of the form $\Sigma^{s\ell}[\cos(\vec{k}_0\vec{x})](\vec{\sigma}\vec{L})$ appears in the mass operator. Thus, Σ ceases to commute not only with the shift and rotation operator, but also with the spin operators $\vec{\sigma}$, although the initial Lagrangian is assumed to be invariant to these operations. It is then necessary to satisfy simultaneously three self-consistency conditions, two of which are similar to (5) and (8), and the third takes the form

$$[\Sigma^{s\ell}, \vec{\sigma}] = U[G, \vec{\sigma}] \tag{9}$$

(a similar condition is obtained also for the "Lane" part of the self-consistent potential $\Sigma_{\Lambda}\vec{\tau}\vec{T}$ in a nucleus at $N \neq Z$ [6]).

We note that the presence of several self-consistency conditions means that there exist in the system several types of collective excitations, the characteristics of which can be determined with the aid of the obtained relations. This problem will be considered in a following paper.

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STATISTICAL BOOTSTRAP AND THE POMERANCHUK MODEL FOR MULTIPLE HADRON PRODUCTION

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We wish to call attention to the profound connection between two statistical hadron models, the Pomeranchuk model [1] and the Hagedorn-Frautschi bootstrap model [2, 3]. In recent papers, Feinberg [4] and Sisakyan, Feinberg, and Chernavskii [5] explained successfully a large number of experimental facts within the framework of the Pomeranchuk model. Similar results are obtained also in the statistical bootsrap model [2, 3]. The key fact here is that the temperature in both models is independent of the system energy. However, the reasons why such a temperature appears in the two models seem to be quite different: in the Pomeranchuk model the cause is the proportionality of the volume of the system in which the thermodynamic equilibrium is established (the Pomeranchuk fireball) to the number n of secondary particles, and in the bootstrap model the cause is the presence of a hadron state density $\rho(m)$ that increases with increasing mass. Moreover, one of the main postulates of the bootstrap model is that the volume of the heavy hadrons (the Hagedorn fireballs) is independent of their mass. All this can give the impression that there is no direct connection between these models. We shall show, however, that in the statistical bootstrap the role of the Pomeranchuk model is played not by the volume of the hadron itself, but by the volume of the system of stable particles (arbitrarily, pions) into which it decays. In accordance with the Pomeranchuk postulate, this volume is proportional to the number of pions, making it possible immediately to establish a correspondence between the different fireball definitions used in the two models.

We start with a relativistically-invariant form of the statistical bootstrap equation, corresponding to the so-called bootstrap condition

$$\rho(m) = d_1\delta(m-m_0) + \sum_{k=2}^{\infty}\left(\frac{V}{8\pi^3}\right)^{k-1}\frac{1}{k!}\prod_{i=1}^{k}\int dm_i^2\,\rho(m_i)\times \int\frac{d^3p_i}{2p_{0i}}\,\delta\Big(m-\sum_{i=1}^{k}p_{0i}\Big)\,\delta^3\Big(\sum_{i=1}^{k}\mathbf{p}_i\Big), \quad (1)$$

where m_0 is the pion mass, V is the hadron volume, d_1 describes the pion degeneracy, and $\rho(m)$ is the hadron density. Using the method of [6, 7], we can obtain an exact solution of (1):

$$\rho(m) = d_1\delta(m-m_0) + \sum_{k=2}^{\infty} d_k\,\theta(m-km_0)\,\mathcal{T}^{(k)}_{(m;\,m_0,\ldots m_0)}\,, \quad (2)$$

where

$$d_k \underset{k\to\infty}{\approx} (d_1 m_0)^k\left(\frac{V}{4\pi^3}\right)^{k-1}\left[2\pi k^3(\ln 4-1)\right]^{-1/2} e^{-k\ln(\ln 4-1)},$$

and

$$\mathcal{T}^{(k)}_{(m;\,m_0,\ldots m_0)} = \prod_{i=1}^{k}\int d^4p_i\;\theta(p_{0i})\,\delta(p_i^2-m_0^2)\,\delta\Big(m-\sum_{i=1}^{k}p_{0i}\Big)\,\delta^3\Big(\sum_{i=1}^{k}\mathbf{p}_i\Big) \quad (3)$$

is the phase volume of k pions connected with a hadron of mass m.