M. A. Troitskii for a useful discussion.

[1] A. A. Grib, E. V. Damaskinskii, and V. M. Maksimov, Usp. Fiz. Nauk 102, 587 (1970) [Sov. Phys.-Usp. 13, 798 (1971)].
[2] V. A. Khodel' and S. A. Fayans, Yad. Fiz. 12, 717 (1970) [Sov. J. Nuc. Phys. 12, 388 (1971)]
[3] N. N. Bogolyubov, JINR Preprint R-1451, Dubna, 1963.
[4] A. B. Migdal, ITF Preprint T-13819, Chernogolovka, 1972.
[5] A. B. Migdal, Teoriya konechnykh fermi-sistem i svoistva atomnykh yader (Theory of Finite Fermi Systems and Properties of Atomic Nuclei), Nauka, 1965; D. F. Zaretski and M. G. Urin, Proceedings of Topical Symposium on Nuclear Physics, Tbilisi, 1967; B. L. Birbrair, Yad. Fiz. 5, 1198 (1967) [Sov.J. Nuc. Phys. 5, 1198 (1967)].

STATISTICAL BOOTSTRAP AND THE POMERANCHUK MODEL FOR MULTIPLE HADRON PRODUCTION

M. I. Gorentshtein, G. M. Zinov'ev, V. A. Miranskii, and V. P. Shelest
Institute of Theoretical Physics, Ukrainian Academy of Sciences

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We wish to call attention to the profound connection between two statistical hadron models, the Pomeranchuk model [1] and the Hagedorn-Frautschi bootstrap model [2, 3]. In recent papers, Feinberg [4] and Sisakyan, Feinberg, and Chernavskii [5] explained successfully a large number of experimental facts within the framework of the Pomeranchuk model. Similar results are obtained also in the statistical bootsrap model [2, 3]. The key fact here is that the temperature in both models is independent of the system energy. However, the reasons why such a temperature appears in the two models seem to be quite different: in the Pomeranchuk model the cause is the proportionality of the volume of the system in which the thermodynamic equilibrium is established (the Pomeranchuk fireball) to the number n of secondary particles, and in the bootstrap model the cause is the presence of a hadron state density $\rho(m)$ that increases with increasing mass. Moreover, one of the main postulates of the bootstrap model is that the volume of the heavy hadrons (the Hagedorn fireballs) is independent of their mass. All this can give the impression that there is no direct connection between these models. We shall show, however, that in the statistical bootstrap the role of the Pomeranchuk model is played not by the volume of the hadron itself, but by the volume of the system of stable particles (arbitrarily, pions) into which it decays. In accordance with the Pomeranchuk postulate, this volume is proportional to the number of pions, making it possible immediately to establish a correspondence between the different fireball definitions used in the two models.

We start with a relativistically-invariant form of the statistical bootstrap equation, corresponding to the so-called bootstrap condition

$$\rho(m) = d_1\delta(m-m_o) + \sum_{k=2}^{\infty}\left(\frac{V}{8\pi^3}\right)^{k-1}\frac{1}{k!}\prod_{i=1}^{k}\int dm_i^2\,\rho(m_i)\times \int\frac{d^3p_i}{2p_{oi}}\,\delta(m-\sum_{i=1}^{k}p_{oi})\,\delta^3(\sum_{i=1}^{k}\mathbf{p}_i), \tag{1}$$

where m_o is the pion mass, V is the hadron volume, d_1 describes the pion degeneracy, and $\rho(m)$ is the hadron density. Using the method of [6, 7], we can obtain an exact solution of (1):

$$\rho(m) = d_1\delta(m-m_o) + \sum_{k=2}^{\infty} d_k\,\theta(m-km_o)\,\mathcal{T}^{(k)}_{(m;\,m_o,\dots m_o)}, \tag{2}$$

where

$$d_k \underset{k\to\infty}{\approx} (d_1m_o)^k\left(\frac{V}{4\pi^3}\right)^{k-1}[2\pi k^3(\ln 4-1)]^{-1/2}\,e^{-k\ln(\ln 4-1)},$$

and

$$\mathcal{T}^{(k)}_{(m;\,m_o,\dots m_o)} = \prod_{i=1}^{k}\int d^4p_i\,\theta(p_{oi})\,\delta(p_i^2-m_o^2)\,\delta(m-\sum_{i=1}^{k}p_{oi})\,\delta^3(\sum_{i=1}^{k}\mathbf{p}_i) \tag{3}$$

is the phase volume of k pions connected with a hadron of mass m.

We can offer the following interpretation of (1) and (2). Divide both halves of (1) by $\rho(m)$. Then the integrals in the right-hand side of (1) specify the average partial widths of resonances of mass m in units in which the average width of resonances having this mass is equal to unity. The fact that the same function $\rho(m)$ is contained in the left and right sides of (1) is a reflection of the following bootstrap condition: the resonances into which the initial resonance decays decay in turn in the very same manner into still lighter hadrons. On the other hand, relation (2) reflects the fact that ultimately all the heavy hadrons should decay into stable pions. It should be assumed here that the n-term in (2) has a dynamic nature and is proportional to the probability of observing n pions at the end of such a decay:

$$d_n \mathcal{T}^{(n)}(m; m_0, \ldots m_0) =$$
$$= \left(\frac{\tilde{V}}{8\pi^3}\right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int dm_i^2\, \delta(m_i - m_0) \int \frac{d^3 p_i}{2p_{0i}}\, \delta\left(m - \sum_{i=1}^{n} p_{0i}\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i\right), \qquad (4)$$

$$\tilde{V} = \left(\frac{d_n n!}{m_0}\right)^{1/(n-1)} \propto Vn. \qquad (5)$$

We recall now that the average multiplicity (n) of the pions in the statistical bootstrap model is proportional to the hadron mass m [3]. Therefore $p_{0\text{eff}}$ will not depend on m in the dominating configurations, and expression (4) can be interpreted as the density of the number of states of a microcanonical ensemble of n free identical particles with volume $V_{\text{eff}} \approx \tilde{V}(m_0/p_{0\text{eff}})$ in their c.m.s. Thus, just as in the Pomeranchuk model, the volume of the pion gas is proportional here to the number of particles n. All the thermodynamic characteristics of the corresponding systems therefore coincide in the two models.

It must be borne in mond, however, that in the statistical bootstrap model, owing to additional dynamic assumptions, there is more information than in the Pomeranchuk model. Thus, it is possible to prove in it the dominant role of the decay of a hadron into a pion and another hadron, and the exponential growth of the hadron density $\rho(m)$ [2, 3]. The predictions of the model coincide only when it comes to describing the decay products at the end of the complete process, and the intermediate stages are not described concretely in any manner in the model [1].

We note in conclusion that, as shown in [8], the statistical approach to dual models leads to results that are close to those obtained within the framework of the statistical bootstrap, so that one cannot exclude the possibility of regarding them as a concrete dynamic realization of either the Hagedorn-Frautschi or the Pomeranchuk model.

[1] I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 78, 889 (1951).
[2] R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).
[3] S. Frautschi, Phys. Rev. 3D, 2821 (1971).
[4] E. L. Feinberg, Usp. Fiz. Nauk 104, 539 (1971)[Sov. Phys.-Usp. 14, 455 (1972)].
[5] I.N. Sisakyan, E. L. Feinberg, and D. S. Chernavskii, Trudy FIAN SSSR 57, 164 (1972).
[6] W. Nahm, Nucl. Phys. B45, 525 (1972)
[7] J. Yellin, Preprint TH-1513, CERN, 1972.
[8] V. A. Miransky, V. P. Shelest, B. V. Struminsky, G. M. Zinovjev, Phys. Lett. 43B, 61 (1973); M. I. Gorenstein, V. A. Miransky, V. P. Shelest, B. V. Struminsky, and G. M. Zinovjev, Preprint ITP-72-168, Kiev, 1972.

VERTEX FUNCTIONS AND POLARIZATION OPERATOR IN (4 - ε)-DIMENSIONAL FIELD THEORY

S. L. Ginzburg
Leningrad Institute of Nuclear Physics, USSR Academy of Sciences

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The vertex functions and the polarization operator in $(4-\varepsilon)$-dimensional field theory with interaction ϕ^4 are calculated for small ε by direct summation of perturbation-theory diagrams. The derived expressions are valid both where perturbation theory is valid and in the scaling region.

As is well known [1], The Landau theory of phase transitions holds in four-dimensional space with logarithmic accuracy. It is therefore natural to expect that in $(4 - \varepsilon)$-dimensional