We can offer the following interpretation of (1) and (2). Divide both halves of (1) by $\rho(m)$. Then the integrals in the right-hand side of (1) specify the average partial widths of resonances of mass m in units in which the average width of resonances having this mass is equal to unity. The fact that the same function $\rho(m)$ is contained in the left and right sides of (1) is a reflection of the following bootstrap condition: the resonances into which the initial resonance decays decay in turn in the very same manner into still lighter hadrons. On the other hand, relation (2) reflects the fact that ultimately all the heavy hadrons should decay into stable pions. It should be assumed here that the n-term in (2) has a dynamic nature and is proportional to the probability of observing n pions at the end of such a decay:

$$d_n\mathcal{T}^{(n)}(m;\, m_o,\, \ldots m_o) =$$

$$= \left(\frac{\tilde{V}}{8\pi^3}\right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int dm_i^2\, \delta(m_i - m_o) \int \frac{d^3p_i}{2p_{oi}}\, \delta\left(m - \sum_{i=1}^{n} p_{oi}\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i\right), \qquad (4)$$

$$\tilde{V} = \left(\frac{d_n n!}{m_o}\right)^{1/(n-1)} \propto Vn. \qquad (5)$$

We recall now that the average multiplicity (n) of the pions in the statistical bootstrap model is proportional to the hadron mass m [3]. Therefore $p_{0\text{eff}}$ will not depend on m in the dominating configurations, and expression (4) can be interpreted as the density of the number of states of a microcanonical ensemble of n free identical particles with volume $\tilde{V}_{\text{eff}} \approx \tilde{V}(m_0/p_{0\text{eff}})$ in their c.m.s. Thus, just as in the Pomeranchuk model, the volume of the pion gas is proportional here to the number of particles n. All the thermodynamic characteristics of the corresponding systems therefore coincide in the two models.

It must be borne in mond, however, that in the statistical bootstrap model, owing to additional dynamic assumptions, there is more information than in the Pomeranchuk model. Thus, it is possible to prove in it the dominant role of the decay of a hadron into a pion and another hadron, and the exponential growth of the hadron density $\rho(m)$ [2, 3]. The predictions of the model coincide only when it comes to describing the decay products at the end of the complete process, and the intermediate stages are not described concretely in any manner in the model [1].

We note in conclusion that, as shown in [8], the statistical approach to dual models leads to results that are close to those obtained within the framework of the statistical bootstrap, so that one cannot exclude the possibility of regarding them as a concrete dynamic realization of either the Hagedorn-Frautschi or the Pomeranchuk model.

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VERTEX FUNCTIONS AND POLARIZATION OPERATOR IN (4 - ε)-DIMENSIONAL FIELD THEORY

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The vertex functions and the polarization operator in $(4-\varepsilon)$-dimensional field theory with interaction ϕ^4 are calculated for small ε by direct summation of perturbation-theory diagrams. The derived expressions are valid both where perturbation theory is valid and in the scaling region.

As is well known [1], The Landau theory of phase transitions holds in four-dimensional space with logarithmic accuracy. It is therefore natural to expect that in $(4 - \varepsilon)$-dimensional space with $\varepsilon \ll 1$ the deviation from this theory would be small. Wilson and Fisher [2] have proposed to use ε as a small parameter for the calculation of the critical exponents. Wilson [3], using renormalization-group ideas, calculated the exponents accurate to ε^3 and obtained an expression for the four-point diagram in the scaling region. Tsuneto and Abrahams [4] did the same by using the Word identity. The field-theory equations, however, were not solved in these papers.

On the other hand, Larkin and Khmel'nitskii [1] have shown that in four-dimensional field theory with interaction ϕ^4 there is realized a logarithmic situation and the main contribution to the vertex functions is made by the so-called parquet diagrams. Naturally, in $(4 - \varepsilon)$-dimensional theory with $\varepsilon \ll 1$ the main contribution should also be made by these diagrams, although the logarithm is replaced by a power-law function with a small exponent ε. The reason is that at small ε a power-law function, like the logarithm, varies little and is large.

In the present paper, by summing parquet diagrams, we obtain in $(4 - \varepsilon)$-dimensional space explicit expressions for the vertex functions and the polarization operator in the entire range of momentum variation, and match the results with those of perturbation theory. We show, in particular, that in the scaling region the coefficient preceding the corresponding degree of the momentum in a vertex with two ends and one corner and in the polarization operator is a power-law function of ε with non-integer exponent. The expression for the four-point diagram goes over in the scaling region into the results of [3, 4].

We consider the theory of an n-component field ϕ_α with interaction $\sum_{\alpha\beta}\phi_\alpha^2\phi_\beta^2$. The zero-order Green's function is equal to $G_0(k) = (r_0 + k^2)^{-1}$, and the unrenormalized vertex is $u_0\Lambda^\varepsilon\delta_{\alpha\beta}\delta_{\mu\nu}$, where Λ is the cutoff momentum and u_0 is a dimensionless parameter, $u_0 \ll 1$.

We consider the vertex part $\Gamma(q_1, q_2, q_3)$ in the case when all the momenta are of the same order. Then Γ is a function of one argument, and by summing the parquet diagrams [1, 5] we obtain for it the following equation:

$$\Gamma_{\alpha\beta\mu\nu}(q) = \Gamma(q)(\delta_{\alpha\beta}\delta_{\mu\nu} + \delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\alpha\nu}\delta_{\beta\mu}),$$
$$\Gamma(q) = u_0\Lambda^\epsilon - (n+8)\int_q^\Lambda \frac{d^dp}{(2\pi)^d} G^2(p)\Gamma^2(p), \tag{1}$$

where d is the dimensionality of space. Assuming that the exponent η vanishes, we put

$$G(p) = p^{-2}, \qquad \frac{d^dp}{(2\pi)^d} \to \mathcal{K}_d\, p^{3-\epsilon}dp,$$

$$\mathcal{K}_d = 2^{-(d-1)}\pi^{-d/2}[\Gamma(d/2)]^{-1}$$

we make the change of variable $y = (\Lambda/q)^\varepsilon$, differentiate with respect to y, and solve the resultant differential equation. We obtain

$$\Gamma(q) = u_0\Lambda^\epsilon t^{-1},$$
$$t = 1 + \frac{n+8}{\epsilon}\mathcal{K}_d u_0\left[\left(\frac{\Lambda}{q}\right)^\epsilon - 1\right]. \tag{2}$$

The equations for the vertex $F_{\alpha\beta}(q)$ represented by diagrams with two ends and one corner and for the polarization operator $\Pi(q)$ take the form [1]

$$F_{\alpha\beta}(q) = \delta_{\alpha\beta} - \int_q^\Lambda \frac{d^dp}{(2\pi)^d} F_{\mu\nu}(p)G^2(p)\Gamma_{\mu\nu\alpha\beta}(p),$$
$$\Pi(q) = \int_q^\Lambda \frac{d^dp}{(2\pi)^d} F_{\alpha\beta}(p)F_{\alpha\beta}(p)G^2(p). \tag{3}$$

Putting $F_{\alpha\beta} = F\delta_{\alpha\beta}$ and solving (3) in the same manner as (1), we obtain

$$F = t^{-(n+2)/(n+8)},$$

$$\Pi = \frac{n}{4-n}\,\frac{1}{v_0 \Lambda^{\epsilon}}\,[\,t^{(4-n)/(n+8)} - 1\,]. \tag{4}$$

From (2) and (4) we obtain at $q \ll \Lambda$, assuming $K_d \approx K_0$

$$\Gamma = \frac{8\pi^2\epsilon}{n+8}\,q^{\epsilon}, \quad F = \left[\frac{\Gamma}{v_0 \Lambda^{\epsilon}}\right]^{(n+2)/(n+8)},$$

$$\Pi = \frac{n}{4-n}\,\frac{1}{v_0 \Lambda^{\epsilon}}\left[\frac{\Gamma}{v_0 \Lambda^{\epsilon}}\right]^{-(4-n)/(n+8)} \tag{5}$$

The first formula in (5) was obtained earlier in [3, 4]. It is seen from (5) that the coefficients preceding the powers of q in F and Π are proportional to $\varepsilon^{(n+2)/(n+8)}$ and $\varepsilon^{-(4-n)/(n+8)}$ and cannot be expanded in powers of ε.

To determine the exponent γ we use the Ward identity [1]:

$$\frac{dr}{dr_0} = F(0), \quad r = G^{-1}(0). \tag{6}$$

Since q goes over into $\sqrt{r}$ as $q \to 0$, we have

$$r \sim \epsilon^{(n+2)/(n+8)}(r_0 - r_{0c})^{\gamma}, \quad \gamma = 1 + \epsilon\frac{(n+2)}{2(n+8)} \tag{7}$$

r_{0c} is the critcal value of r_0. The expression for was derived earlier in [3, 4]. The specific heat c is proportional to $\Pi(0)$, and therefore

$$c \sim \epsilon^{-(4-n)/(n+8)}(r_0 - r_{0c})^{-\epsilon\frac{4-n}{2(n+8)}}. \tag{8}$$

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NOTE

A footnote was left out in connection with the title of the article by G. A. Askaryan, V. A. Namion, and M. S. Rabinovich, Vol. 17, No. 10, p. 424. The footnote reads: "This article was reported by the authors at the Physics Institute of the USSR Academy of Sciences in July 1972, at which time copies of it were distributed."