excess neutrons fill the states above the doubly-magic "82-82" core, without essentially perturbing it, and lead to an increase of R. The obtained value of β_3 agrees with the data on inelastic proton and electron scattering at low energies: β_3 = 0.11 [10], 0.14 [8], 0.107 [6], and 0.12 [11]. It is interesting to note that the course of the cross sections of Ni and Nb agrees with the Blair's phase rule [12], namely, the elastic and inelastic cross sections vary in phase for transitions with change of parity (3^- in Pb), and in counterphase for transitions without change of parity (2^+ in Ni).

We note in conclusion that the oberved good agreement between theory and experiments proves that the employed approach is adequate. Although the collective states are described phenomenologically, the parameters that enter in the theory are compared simultaneously with data on elastic and and inelas tic scattering of protons, and also of electrons where possible (Ni). This comparison increases the reliability not only of the parameters themselves, but also of the method.

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SHELL EFFECTS IN ELASTIC SCATTERING OF NUCLEONS BY NUCLEI, AND NEUTRON STRENGTH FUNCTIONS

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The shell approach to the description of nucleon scattering by nuclei is used to parametrize the scattering matrix averaged over the resonances of the compound nucleus. Concrete calculations are performed for the s-neutron strength functions. It is shown that, in contradiction to the predictions of the optical model, the magnitude and the isotopic dependence of the strength function at the minimum ($A \sim 100 - 140$) are determined, in the main, by the spectroscopic factor of the 3s state.

The phenomenological description of the scattering of nucleons by nuclei is based on the use of the optical model. In this model, the excitation of the compound-nucleus levels during the scattering process is taken into account by introducing an imaginary part in the optical potential. The parameters of this part are chosen to fit best the experimental data. A more consistent approach to the description of the excitation of the compound-nucleus levels is based on the use of the shell model [1, 2]. Within the framework of the shell approach to the theory of nuclear reactions it is also possible to calculate the average S matrix ($\bar{S}$) and to trace its dependence on the shell-model parameters and on the quantities that determine the binding force of the single-particle states with the compound-nucleus levels. Since, unlike the optical model, the shell approach takes into account effects connected with the existence of Fermi filling of the single-particle states, the average S matrix can depend also on the structure of the Fermi boundary.

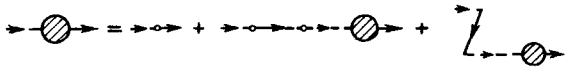
In the shell model, the wave functions of the excited states of the nuclei (systems of A + 1 nucleons) can be classified in accordance with the number of particles and holes. In addition to the single-particle states of the continuous and discrete spectra $|1p\rangle \equiv |E\rangle$, there exist also multiparticle configurations of the type 2p - 1h, 3p - 2h, etc. As a rule, at excitation energies on the order of or larger than the binding energies of the nucleon, the multi-

particle configurations with given spin and parity are not isolated (with the possible exception of light and magic nuclei). It is therefore convenient first to diagonalize the shell-model Hamiltonian on the basis of the multiparticle configurations. This results in compound-nucleus states $|\lambda\rangle$ that constitute superpositions of multiparticle configurations, the simplest of which are configurations of the type 2p - 1h. In this basis, the solution of the problem of nucleon scattering reduces to the determination of the wave functions of the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{F}, \tag{1}$$

where $\hat{H}_0|\lambda\rangle = E_\lambda|\lambda\rangle$ and $\hat{H}_0|1p\rangle = E|1p\rangle$, and the only nonzero matrix elements of the residual-interaction operator are those for the transitions $|\lambda\rangle \leftrightharpoons |1p\rangle$ and $|0\rangle \leftrightharpoons |1h, \lambda\rangle$. Here $|0\rangle$ and $|1h\rangle$ denote respectively the wave functions of the ground state of the target nucleus (vacuum) and of the hole state.

Just as before [2], we use the equations of the theory of quantum transitions to describe the resonant scattering. We introduce the quantities $V_{\lambda/E}$, which have the meaning of the probability amplitudes of the transitions $|\lambda\rangle \leftrightharpoons |E\rangle$. The graphic form of the equation for these amplitudes is



(2)

The dashed lines correspond here to the states $|\lambda\rangle$, and the solid ones to the states $|1p\rangle$ and $|1h\rangle$. The first term in (2) denotes the matrix element $F_{\lambda/E}$; the summation in the second term is over the single-particle states with energy higher than the Fermi energy; the third term takes into account the possibility of the virtual production of the states $|1h\rangle$ and $|\lambda\rangle$ from vacuum. The analytic form of (2) is

$$U_{\lambda/E} = F_{\lambda/E} + \sum_{\lambda'} \Pi_{\lambda\lambda'}(E - E_{\lambda'})^{-1} U_{\lambda'/E};$$
$$\Pi_{\lambda\lambda'}(E) = \sum_{E'} F_{\lambda/E'} \frac{1 - 2n_{\epsilon'}}{E - E' + i\epsilon} F_{E'/\lambda'}, \tag{3}$$

where $n_{\epsilon'}$ are the occupation numbers (the spectroscopic factor for the target nucleus). The summation in the expression for $\Pi_{\lambda\lambda'}$ is carried out over all the single-particle states of the shell Hamiltonian, including the continuous spectrum.

The amplitudes $U_{\lambda/E}$ determine the diagonal elements of the scattering matrix in accordance with the relations [1, 2]

$$S = e^{2i\delta} - i2\pi\rho_E \sum_{\lambda} F_{E/\lambda}(E - E_\lambda)^{-1} U_{\lambda/E}; \quad \bar{S}(E) = S(E + iI), \tag{4}$$

where $\delta(E)$ is the phase shift of the potential scattering, ρ_E is the density of the single-particle states in the continuous spectrum, and I is the averaging interval.

The solution of (3) can be obtained with the aid of the Green's function of the radial Schrödinger equation for the shell-model potential. The average S matrix obtained from (3) and (4) coincides with the optical-model S matrix (S_{opt}) only if no account is taken of the influence of the Fermi occupation ($n_E \equiv 0$). Near the single-particle resonances (shape resonances) S also coincides with S_{opt}, for in this case the contributions of the other single-particle resonances and bound states can be neglected. Far from shape resonances, owing to the summation in (3) over single-particle states (with given spin and parity values) having an energy lower than the Fermi energy, S(E) differs from $S_{opt}(E)$. This difference, as follows from (3), is determined by the parameter $W(E)n_{E_b}/(E + E_b)$, where W(E) is the imaginary part of the optical potential and E_b is the binding energy of the single-particle level closest to the Fermi boundary.

The influence of the structure of the Fermi boundary is manifest, in particular, in the value of the strength function for slow neutrons, which is determined by the relation $SF = (2\pi)^{-1}(1 - |\bar{S}|^2)$. The largest difference between SF and $(SF)_{opt}$ should be expected in the atomic-weight region where the corresponding single-particle level crosses the Fermi boundary.

To calculate SF we assume that W is constant over the volume of the nucleus. In this case one can obtain a nalytic solution of (3) with accuracy $\sim A^{-1/3}$. As a result, SF can be calculated with the aid of the optical model with the potential

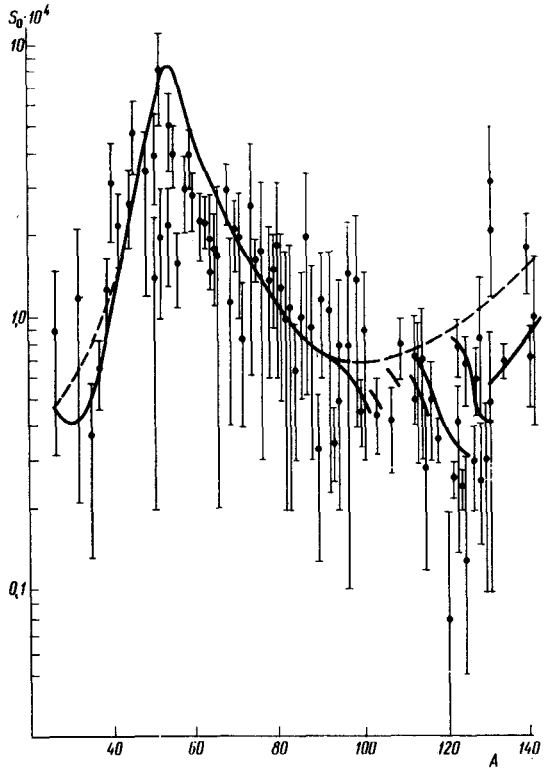


Fig. 1

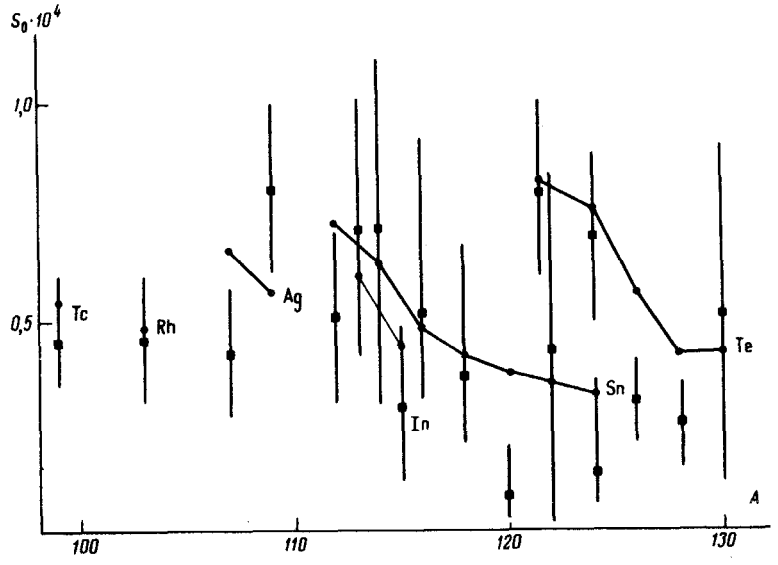


Fig. 2

$$V_{opt} = V_{sh} + \delta V; \quad \delta V = (-iW + \Delta)\left(1 - \sum_b \frac{2n_b P_b^2 (-iW + \Delta)}{E + E_b + 2n_b (-iW + \Delta)}\right),$$

where Δ is the change of the real part of the optical potential in comparison with the shell value, and P_b is the overlap integral of the wave functions of the continuous and discrete spectra, normalized to unity in the volume of the nucleus.

The s-neutron strength functions were calculated for approximately 100 spherical nuclei in the range of A from 27 to 142. The shell-potential parameters were taken from [3], and the spectroscopic factors for the 3s states were taken from [4]. The parameters W and Δ were obtained by least squares from the condition of best agreement between the experimental and calculated values of the strength functions. These parameters turned out to equal W = 3.5 MeV and Δ = 1.5 MeV. Figure 1 shows, together with the experimental data, the calculated relative strength functions $S_0 = E^{-1/2}SF$ (E in eV) on the basis of the shell approach (solid line) and in accordance with the optical model with the indicated values of W and Δ (dashed line). As expectted, the largest difference between SF and $(SF)_{opt}$ is reached in the A range from 100 to 140, where the single-particle 3s level crosses the Fermi boundary. In this region, the strength functions are most sensitive to the value of the spectroscopic factor, the variations of which determine, in the main, the isotopic course of SF. As follows from Fig. 2, this conclusion agrees with the experimental data.

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