At $T \lesssim E_b$, observation of this band should make it possible to determine E_b, and owing to the high values of f_b such measurements are possible at relatively low exciton concentrations. A comparison with the luminescence spectra should make it possible to separate reliably the biexciton spectrum.

3. There is no analogous effect for indirect transitions. In crystals with indirect transitions, however, there can exist additional extrema corresponding to direct transitions, as is the case with Ge. Photoproduction of a direct exciton near a thermalized indirect one is then possible, and the two can form a biexciton. The oscillator strength of such a transition will also be gigantic and described by a formula that differs only slightly from (2) (mainly because of the difference between the exciton masses).

The situation should be particularly interesting if a confirmation is found for the polyexciton idea [2], according to which the binding energy in multivalley crystals increases progressively with increasing the number N of the excitons contained in the crystal. In this case the width of the absorption band corresponding to the "completion" of the polyexciton on account of the direct exciton, should be small ($\sim N^{-1}$) owing to the large mass of the polyexciton.

Thus, in crystals with indirect transitions one can expect to observe aggregates that do not appear under quasiequilibrium conditions. Since drops cannot generate the structural spectra considered here, these spectra can be useful for the study of the composition of a quasi-equilibrium phase.

4. We note in conclusion that in addition to the absorption connected with transitions to a discrete biexciton level, there exists an absorption corresponding to transitions to the continuous spectrum of a system of two excitons. It also contains a large factor $(\kappa^3 v)^{-1} \gg 1$, and furthermore diverges like $[\omega - (E_g - R)]^{-2}$ near the frequency corresponding to the production of a free exciton (on both sides of this frequency). Although the divergence is cut off by the exciton damping and by the polariton effect, its intensity is nevertheless large. Equally large is the probability of the inverse process, collision emission.

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0^+ STATE OF ALPHA PARTICLE IN A MODEL WITH SEPARABLE POTENTIAL

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We consider in this article a new approach to the four-nucleon problem, based on the use of multiparticle integral equations. This approach is free of the shortcomings of the usual variational methods [1] and is equally applicable to the calculation of discrete and continuous spectra. In particular, it is easy to obtain in this method practically exact results for states of the cluster type, for which the method of hyperspherical functions [2] results in worse convergence and needs to be modified [3]. We present below the first calculations of this kind for 0^+ states of an α particle with allowance for the spin dependence of the nucleon-nucleon interaction. It must be emphasized that allowance for spin effects makes the solution method much more complicated in comparison with the earlier "spinless" calculations of few-nucleon systems. On the other hand, if realistic nucleon-nucleon potentials are used there are no grounds whatever for neglecting forces that depend on the spin orientations. We have considered the variant of integral equations of the Yakubovskii type [4], which take the form of two-dimensional integral equations if a separable representation is used for the two-particle T matrix [5, 6]. One of the methods of solving equations of this kind is the method of separable approximation of the amplitudes for the subsystems 3*1 and 2*2, in terms of which the kernels of the four-particle equations are expressed.

The separable approximation, which makes it possible to reduce the equations to a one-dimensional form, can be effected by many means. We use here a method based on an expansion of the Hilbert-Schmidt type. We shall show that this method makes it possible to obtain even in the first approximation, with an accuracy on the order of several per cent, the binding energy of the ground state. Another important advantage of the method is that it can greatly decrease the number of equations to be solved. The Hilbert-Schmidt method, which is well known as applied to the two-particle amplitude [7], was used successfully to solve three-particle equations [8]. A generalization of the method for three-particle amplitudes was obtained in [9]; similar results are obtained also for the 2*2 amplitudes.

To illustrate the convergence of the method, we consider first the simplest case of identical bosons [10] interacting via a triplet separable potential of the Yamaguchi type [11]. We use the notation of [6, 9], the parameters of the potentials are given in [9], and the constant $\lambda/2m$, where m is the nucleon mass, is equal to 20.73622 MeV-F^2.

The form factor in a system of four identical particles is expressed in terms of the two functions $A(\vec{k}, \vec{p}, \vec{q}|z)$ and $B(\vec{k}, \vec{p}, \vec{q}|z)$. For a separable interaction, we can separate the dependence on k in the functions A and B:

$$A(k, p, q|z) = \sqrt{\frac{\lambda}{2m}}\,\frac{g(k)\,a(p, q|z)}{d\left(z - \frac{p^2}{2m} - \frac{q^2}{2m}\right)}$$

$$B(k, p, q|z) = \sqrt{\frac{\lambda}{2m}}\,\frac{g(k)\,b(p, q|z)}{d\left(z - \frac{p^2}{2m} - \frac{q^2}{2m}\right)}. \tag{1}$$

For the state $L^P = 0^+$, the function $b(p, q|z)$ depends only on the moduli of the vectors p and q. We neglect the contribution of the three-particle configurations with $\ell \neq 0$, in which case the function $a(p, q|z)$ is likewise independent of the angle variables. We introduce in addition the eigenfunctions $w_m(p|z)$ $[v_m(p|z)]$ and the eigenvalues $\eta_m(z)$ $[\xi_m(z)]$ for the 3*1 and 2*2 amplitudes. For the 3*1 subsystems, these quantities are given by formulas (8, 10) of [9], and the quantities v_m and ξ_m are analogously defined. We expand the form factors a and b in series in the eigenfunctions w_m and v_m:

$$a(p, q|z) = \Sigma w_m\left(p\,|\,z - \frac{q^2}{2m}\right)a_m(q|z),\quad b(p, q|z) = \sum_m v_m\left(p\,|\,z - \frac{q^2}{2m}\right)b_m(q|z). \tag{2}$$

Using the fact that w_m and v_m are orthonormalized, we obtain a system of equations for the functions a_m and b_m. In accordance with the general structure of the Yakubovskii equations, this system can be written for a_m only, and the functions b_m can be expressed in terms of a_m by means of an integral operator. The homogeneous system with eigenvalue $\mu(z)$ is given by[1)]

$$a_m(q|z) = \mu^{-1}(z)\left\{\Phi_m\left(z - \frac{q^2}{2m}\right)\sum_n \int K_{mn}(q, q'|z)\,a_n(q'|z)\,q'^2dq'\right\}, \tag{3}$$

where

$$K_{mn}(q, q'|z) = C_{mn}(q, q'|z) + \sum_k \int D_{mk}(q, q''|z)\,\Psi_k\left(z - \frac{q''^2}{2m}\right)D_{nk} \times (q', q''|z)\,q''^2dq'', \tag{4}$$

1) The discrete levels z_i are determined from the condition $\mu_i(z_i) = 1$.

$$\Phi_m(z) = \eta_m(z)/[1-\eta_m(z)], \quad \Psi_m(z) = \xi_m(z)/[1-\xi_m(z)], \tag{5}$$

$$C_{mn}(q, q'|z) = \frac{1}{2}\left(\frac{3}{2\sqrt{2}}\right)^3 \int_{-1}^{1} w_m\left(Q_1 | z - \frac{q^2}{2m}\right) w_n\left(Q_2 | z - \frac{q'^2}{2m}\right) \times \tag{6}$$

$$\times\, d^{-1}\left[z - \frac{1}{2m}(q^2 + Q_1^2)\right] dx$$

$$D_{mn}(q, q'|z) = \frac{1}{2}\left(\sqrt{\frac{3}{2}}\right)^3 \int_{-1}^{1} w_m\left(S_1|z - \frac{q^2}{2m}\right) v_n\left(S_2|z - \frac{q'^2}{2m}\right) \times \tag{7}$$

$$\times\, d^{-1}\left[z - \frac{1}{2m}(q^2 + S_1^2)\right] dx\,.$$

The multiplicity of the system (3) is equal to the number N_η of the three-particle subsystem eigenvalues that are taken into account, while the number N_ξ determines the accuracy with which the kernel is calculated. Table I lists the calculated energies of the ground (z_1) and excited (z_2) states for different values of N_η and N_ξ.

Table I. 0^+ states of four identical bosons.

N_η	N_ζ	z_1, MeV	z_2, MeV
1	1	−87.67	–
2	2	−90.01	−25.98
3	3	−90.09	−26.48
4	4	−90.10	−26.64

We note that even the first approximation $N_\eta = N_\xi = 1$ yields the binding energy of the ground state with accuracy ∼3%, while the approximation $N_\eta = N_\xi = 2$ gives the binding energy of the excited state with the same accuracy. Our final result, $z_1 = -90.10$ MeV and $z_2 = -26.64$ MeV, should be compared with the reuslts of [10], $z_1 = -84.66$ MeV and $z_2 = -24.87$ MeV (we recall that the Bateman method with two separable terms, used in [10], also overestimates the first two-particle threshold, giving $z_0 = -24.55$ MeV instead of $z_0 = -25.56$ MeV). We see that the results agree qualitatively, but the Hilbert-Schmidt method is more accurate.

We now take into account the spin and isospin dependence of the nucleon-nucleon forces. We chose the triplet ($i = 0$) and singlet ($i = 1$) interaction in the form of a separable s-wave Yamaguchi potential. The functions A in (1) are characterized, besides the total spin S and isospin T, also by the values of the spin σ and isospin τ of the separated triad of particles, as well as by the isospin i of the separated pair of particles. The functions B are likewise characterized, besides S and T, by the isospin values i and j of two particle pairs. The general results for different values of S and T were obained in [6]. We consider below the state $S = T = 0$. In this case $\sigma = \tau = 1/2$, and the indices i and j of the function B take on identical values $i = j = 0$ and $i = j = 1$. We thus have two functions $A_i(k, p, q|z)$ and two functions $B_i(k, p, q|z)$. Putting $A_i = \sqrt{\lambda_i/2m}(g_i a_i/d_i)$ and $B_i = \sqrt{\lambda_i/2m}(g_i b_i/d_i)$, we write down an expansion of type (2) in the form

$$\begin{aligned} a_i(p, q|z) &= \sum_m w_{im}\left(p|z - \frac{q^2}{2m}\right) a_m(q|z)\,, \\ b_i(p, q|z) &= \sum_m v_{im}\left(p|z - \frac{q^2}{2m}\right) b_{im}(q|z)\,, \end{aligned} \tag{8}$$

where w_{0m} and w_{1m} are a pair of three-particle eigenfunction with quantum numbers $\sigma = \tau = 1/2$, corresponding to the eigenvalues $\eta_m^{1/2\ 1/2} \equiv \eta_m$ [see [9], formulas (18, 19)], and v_{0m} [v_{1m}] are the eigenfunctions for the 2*2 channel, corresponding to the eigenvalues ξ_{0m} [ξ_{1m}].

Table 2. 0^+ states of α particle.

$N_\eta^{(+)}$	$N_\eta^{(-)}$	N_ζ	z_1, MeV	z_2, MeV
1	0	1	- 44.29	—
1	0	4	- 44.54	- 11.06
2	1	4	- 45.69	- 11.39
3	2	4	- 45.73	- 11.63
4	3	4	- 45.73	- 11.69

The system of equations for the functions a_m again takes the form (3), where

$$K_{mn}(q, q'|z) = \sum_i \left\{ C^i_{mn}(q, q'|z) + \sum_k \int D^i_{mk}(q, q''|z) \times \right. \\ \left. \times \Psi^i_k \left(z - \frac{q''^2}{2m}\right) D^i_{nk}(q', q''|z)\, q''^2 dq'' \right., \tag{9}$$

and the quantities Ψ^i_k, C^i_{mn}, and D^i_{mn} are obtained from formulas (5) - (7) in terms of ξ_{im}, w_{im}, and v_{im}, respectively. The results of the calculations are given in Table 2, where the symbols $N_\eta^{(+)}$ and $N_\eta^{(-)}$ denote the numbers of the positive and negative eigenvalues for the three-particle problem[1)].

Our final result is $z_1 = -45.73$ MeV and $z_2 = -11.69$ MeV. Subtracting the threshold energy ($z_0 = -11.03$ MeV) we obtain $E_1 = z_1 - z_0 = -34.70$ MeV and $E_2 = z_2 - z_0 = -0.66$ MeV. The experimental values, reckoned from the binding energy of tritium (-8.484 MeV), are $E_1 = -19.840$ MeV and $E_2 = +0.4$ MeV [12]. We see thus that the Yamaguchi potential overbinds the ground state of a four-particle system much more strongly than that of the three-particle state (the overbinding for the triton is 2.55 MeV). It is quite remarkable, however, that in our calculations, both for identical bosons and for spin-dependent forces, the excited 0^+ state lies near the pT threshold, as is observed in experiment (the ground states for these cases differ by almost 45 MeV!). This circumstance calls for an additional theoretical study.

We note in conclusion that our exact results are close to those obtained in the cluster approximation 4 = 3*1 [13] in equations of the Omnes type [13]. In this approximation we obtained z_1(Omnes) = - 39.6 MeV (the excited 0^+ state is located in this case near the pT threshold). This figure differs from the exact value by only 10%. A similar approximation in the Yakubovskii equations yields z_1(Yakubovskii) = -26.24 MeV. The excited 0^+ level does not appear in this case. We see thus that the contribution of the 3*1 channel in the Omnes and Yakubovskii equations turns out to be quite different. The latter is not surprising, since the subclasses of the perturbation-theory diagrams summed in this cluster approximation in the Omnes and Yakubovskii equations are also quite different.

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USE OF SECOND SOUND TO INVESTIGATE THE INHOMOGENEOUS DENSITY DISTRIBUTION OF THE SUPERFLUID PART OF HELIUM II NEAR THE λ POINT

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One can hope to investigate the density distribution of the superfluid part of helium II near the λ point by "sounding" the helium (in particular, the boundary between helium I and helium II in a gravitational or electric field) by means of second-sound waves.

1. Interest in the study of the properties of He-II near the λ point has strongly increased of late. There is no doubt that in many problems it is necessary to take into account the spatial inhomogeneity of the density of the superfluid part of the liquid, $\rho_s = m|\Psi|^2$, with the ordering parameter Ψ determined from the corresponding equations (see [1 - 5] and the literature cited in [4, 5]). In the absence of an electric or gravitational external field, the characteristic dimension of the inhomogeneity of ρ_s is $\xi(T) = \hbar/\sqrt{2mA(T)}$, where A(T) is the coefficient of $|\Psi|^2$ in the expansion of the thermodynamic potential $\Omega(\mu, T, |\Psi|^2)$ in terms of $|\Psi|^2$. The length $\xi(T) = \xi_{0M}(\Delta T)^{-2/3} \sim 3\times10^{-4}$ cm even at $\Delta T = T_\lambda - T \sim 10^{-6}$ °K (here and throughout we use the phenomenological-expansion parameters calculated in [2]; see also [5]). The experimental study of the inhomogeneity of ρ_s is therefore far from being a simple matter, even though it can be performed by a number of methods [6 - 9]. The measurement possibilities become greater if the He is in a gravitational [5, 10] or in an electric [5] field. The point is that in an external field with a slowly varying potential U(x) the boundary between the normal and the superfluid phases of the helium is smeared out, and the characteristic length ℓ, which determines the variation of ρ_s in the transition layer, is given by (see [5] for details)

$$\ell = \xi_{0M}^{3/5}\left(\left|\frac{dT_\lambda}{d\mu}\right| \left|\frac{dU}{dx}\right|\right)^{-2/5},$$

where $dT_\lambda/d\mu$ is the slope of the λ curve, and the derivative dU/dx is taken on a line on which ρ_s vanishes of the gradient term is disregarded in the expansion of Ω. In a gravitational field $|dV/dx| = g$ and $\ell_g = \xi_{0M}^{3/5}(|dT_\lambda/d\mu|g)^{-2/5} = 6.8\times10^{-3}$ cm. In the electric field of a charged filament, $|dV/dx|^{\varepsilon} = (2\Delta T/|dT_\lambda/d\mu|)^{3/2}(\alpha E^2(R)R^2)^{-1/2}$ and $\ell_E = \xi^{3/5}(T)(|dT_\lambda/d\mu|\alpha E^2(R)R^2/8\Delta T)^{1/5}$. Here R is the radius of the filament, E(R) is the field intensity on the surface of the filament, and $\alpha = 3.1\times10^{-2}$ cm^3/g is the polarizability per unit mass of helium. Hence $\ell_E \sim 3\times10^{-2}$ cm at $R \sim 1$ cm, $\Delta T \sim 10^{-6}$ °K, and $E(R) \sim E_{breakdown} \sim 2\times10^6$ V/cm. Under such conditions one can hope to investigate the dependence of $\rho_s(T, r)$ on the coordinate $\vec{r}$ by sounding the distribution