[5] V. F. Kharchenko and V. E. Kuzmichev, Nucl. Phys. A183, 106 (1972). EA196 636 (1972).
[6] I. M. Narodetskii and I. L. Grach, Yad. Fiz. 18, No. 2 (1973) [Sov. J. Nuc. Phys. 18, No. 2, (1974)].
[7] S. Weinberg, Phys. Rev. 131, 440 (1963). L. D. Faddeev, Proc. Fifth Internat. Conf. on the Physics of Electronic and Atomic Collisions, S. H. Branscomb, ed. (boulder, Colorado, 1968), p. 145. I. M. Narodetskii, Yad. Fiz. 9, 1086 (1969) [Sov. J. Nuc. Phys. 9, 636 (1969)].
[8] A. G. Sitenko and V. F. Kharchenko, Usp. Fiz. Nauk 103, 469 (1971) [Sov. Phys.-Usp. 14, 125 (1971)].
[9] I. M. Narodetskii, E. S. Gal'pern, and V. N. Lyakhovitskii, Yad. Fiz. 16, 707 (1972) [Sov. J. Nuc. Phys. 16, 395 (1973)]. ZhETF Pis. Red. 15, 544 (1972) [JETP Lett. 15, 385 (1972)].
[10] V. F. Kharchenko and V. E. Kuzmichev, Phys. Lett. 42B, 328 (1972).
[11] Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).
[12] W. Meyerhof and T. Tombrello, Nucl. Phys. A109, 1 (1968).
[13] I. M. Narodetskii, V. N. Lyakhovitskii, and E. S. Gal'pern, ZhETF Pis. Red. 16, 431 (1972) [JETP Lett 16, 306 (1972)]. I. M. Narodetskii, Yad. Fiz. 18, 67 (1973) [Sov. J. Nuc. Phys. 18, No. 1 (1974)].
[14] R. Omnes, Phys. Rev. B165, 1265 (1968). I. M. Narodetskii and O. A. Yakubovskii, Yad. Fiz. 14, 315 (1971) [Sov. J. Nuc. Phys. 14, 178 (1972)].

USE OF SECOND SOUND TO INVESTIGATE THE INHOMOGENEOUS DENSITY DISTRIBUTION OF THE SUPERFLUID PART OF HELIUM II NEAR THE λ POINT

V. L. Ginzburg and A. A. Sobyanin
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 25 May 1973
ZhETF Pis. Red. 17, No. 12, 698 - 702 (20 June 1973)

One can hope to investigate the density distribution of the superfluid part of helium II near the λ point by "sounding" the helium (in particular, the boundary between helium I and helium II in a gravitational or electric field) by means of second-sound waves.

1. Interest in the study of the properties of He-II near the λ point has strongly increased of late. There is no doubt that in many problems it is necessary to take into account the spatial inhomogeneity of the density of the superfluid part of the liquid, $\rho_s = m|\Psi|^2$, with the ordering parameter Ψ determined from the corresponding equations (see [1 - 5] and the literature cited in [4, 5]). In the absence of an electric or gravitational external field, the characteristic dimension of the inhomogeneity of ρ_s is $\xi(T) = \hbar/\sqrt{2mA(T)}$, where $A(T)$ is the coefficient of $|\Psi|^2$ in the expansion of the thermodynamic potential $\Omega(\mu, T, |\Psi|^2)$ in terms of $|\Psi|^2$. The length $\xi(T) = \xi_{0M}(\Delta T)^{-2/3} \sim 3\times10^{-4}$ cm even at $\Delta T = T_\lambda - T \sim 10^{-6}$ °K (here and throughout we use the phenomenological-expansion parameters calculated in [2]; see also [5]). The experimental study of the inhomogeneity of ρ_s is therefore far from being a simple matter, even though it can be performed by a number of methods [6 - 9]. The measurement possibilities become greater if the He is in a gravitational [5, 10] or in an electric [5] field. The point is that in an external field with a slowly varying potential $U(x)$ the boundary between the normal and the superfluid phases of the helium is smeared out, and the characteristic length ℓ, which determines the variation of ρ_s in the transition layer, is given by (see [5] for details)

$$\ell = \xi_{0M}^{3/5}\left(\left|\frac{dT_\lambda}{d\mu}\right|\left|\frac{dU}{dx}\right|\right)^{-2/5},$$

where $dT_\lambda/d\mu$ is the slope of the λ curve, and the derivative dU/dx is taken on a line on which ρ_s vanishes of the gradient term is disregarded in the expansion of Ω. In a gravitational field $|dV/dx| = g$ and $\ell_g = \xi_{0M}^{3/5}(|dT_\lambda/d\mu|g)^{-2/5} = 6.8\times10^{-3}$ cm. In the electric field of a charged filament, $|dV/dx|_E = (2\Delta T/|dT_\lambda/d\mu|)^{3/2}(\alpha E^2(R)R^2)^{-1/2}$ and $\ell_E = \xi^{3/5}(T)(|dT_\lambda/d\mu|\alpha E^2(R)R^2/8\Delta T)^{1/5}$. Here R is the radius of the filament, E(R) is the field intensity on the surface of the filament, and $\alpha = 3.1\times10^{-2}$ cm³/g is the polarizability per unit mass of helium. Hence $\ell_E \sim 3\times10^{-2}$ cm at R ∼ 1 cm, ΔT ∼ 10^{-6} °K, and $E(R) \sim E_{breakdown} \sim 2\times10^6$ V/cm. Under such conditions one can hope to investigate the dependence of $\rho_s(T, r)$ on the coordinate $\vec{r}$ by sounding the distribution of ρ_s with second sound. The present note is devoted to a discussion of this possibility[1)].

In the general case, to solve the problem of propagation and transformation of sound waves in a medium with spatially-inhomogeneous ρ_s it is necessary to consider the complete system of hydrodynamic equations for He-II near the λ point [11, 12]. It is important to bear in mind here that second sound can be transformed not only into first sound, but also into a heat-conduction wave. The corresponding group of problems, as applied to the reflection of sound from a solid wall and its scattering by small particles of ions, is of great interest but has never been discussed. We confine ourselves here to a simpler situation, when the geometrical-optics approximation can be used to analyze the wave propagation, and effects of absorption and dispersion of the sound can be neglected.

Specifically, we consider normal incidence of a plane wave of second sound on an He-I - He-II interface in a gravitational field. For geometrical optics to be applicable and for the absorption and dispersion effects to be small, the wave frequency ω should satisfy the conditions

$$c_2(x)/\gamma\xi(x) > \omega > \beta|dc_2(x)/dx|_{max}, \tag{1}$$

where $c_2(x) = [T\sigma^2\rho_s(x)/\rho C_P]^{1/2}$ is the speed of the second sound, $\xi(x) = \sqrt{\hbar^2/4m^2\rho_s(x)(\partial^2\Omega/\partial\rho_s^2)_{\sigma,P}}$ is the coherence length, and β and γ are certain coefficients, with $\beta \sim 1$ while γ is determined by the character of the growth of the sound absorption at high frequencies. Within the framework of the equations of [11], γ is independent of x, and within the framework of the system of [12] $\gamma \sim \rho_s^{1/2}(x)$. Experimentally [13], at any rate, $\gamma < 1$. In terms of the dimensionless variables of [5]

$$u = x/\ell_g \text{ and } \eta^2 = \rho_s/\rho_{sg} \text{ where } \rho_{sg} = 1{,}43\,\rho_\lambda(\xi_{OM}/\ell_g) \tag{2}$$

we obtain

$$\begin{aligned} c_2(x) &= c_{20}(\xi_{OM}/\ell_g)^{1/2}\eta(u), \quad c_{20} = 1{,}8\cdot 10^3 \text{ cm/sec} \\ \xi(x) &= \xi_0(\ell_g/\xi_{OM})\eta^{-1}(u)u^{-1/3}, \quad \xi_0 = 1{,}85\,\text{Å} \end{aligned} \tag{3}$$

and the inequality (1) takes the form

$$\frac{1}{\gamma}\,\frac{c_{20}}{\xi_0}\left(\frac{\xi_{OM}}{\ell_g}\right)^{3/2}\eta^2(u)u^{1/3} > \omega > \beta\frac{c_{20}}{\xi_{OM}}\left(\frac{\xi_{OM}}{l_g}\right)^{3/2}\left|\frac{d\eta}{du}\right|_{max}. \tag{4}$$

The profile of $\eta(u)$ was calculated in [5]. Using the plot and the asymptotic formula given there for $\eta(u)$ in the region of the "normal" phase, we can easily find that under the most favorable choice of the frequency

$$\omega \sim \beta\frac{c_{20}}{\xi_{OM}}\left(\frac{\xi_{OM}}{l_g}\right)^{3/2}\left|\frac{d\eta}{du}\right|_{max} \sim 200\ \text{sec}^{-1}$$

the penetration of the second sound into the interior of the normal phase (u < 0 is possible up to distances

$$|u| \lesssim u_c \sim \left[\frac{5}{6}\ln\left(\frac{\xi_{OM}\,3^{1/5}e^{-2/5}}{\xi_0\,\gamma\beta}\right)\right]^{3/5}$$

1) The use of light, x-rays, and neutrons for this purpose is, generally speaking, much less promising, by virtue of the low polarizability of liquid helium and the weak dependence of the total density ρ on ρ_s near the λ point. It would be of definite interest, however, to observe diffraction of x-rays or even light by a grating of vortices in rotating helium under pressure (near the melting point), for in this case the core of the vortex should become solid.

If $\gamma\beta \lesssim 0.1$, then $u_c \gtrsim 1.5$, and the phase shift (in the region of the "tail" of the $\eta(u)$ distribution),

$$\eta(u)\ ),\ \ \Delta\phi = \beta \left|\frac{d\eta}{du}\right|_{max} \int_{u_c}^{0} \frac{du}{\eta(u)},$$

which depends exponentially on u_c, is quite large ($\Delta\phi \gtrsim 2\pi$).

To measure the phase shift $\Delta\phi$ (or the pulse delay time $\Delta t = \Delta\phi/\omega$) it is evidently necessary to place at a distance $|u| < u_c$ a certain wall and to observe the reflection of second sound from it. The propagation of the sound near the wall, as already mentioned, has not yet been considered in detail. There are grounds, however, for assuming that in the region near the wall where the geometrical-optics approximation can not be applied to second sound, the order of magnitude of the additional phase shift does not exceed 2π and, more importantly, changes little when the wall is displaced, provided the thermal resistance of the wall is large.

Everything said above applies equally well to an electric field, except that the characteristic scale ℓ_g must be replaced by ℓ_E. We have here an additional possibility of varying the width of the separation boundary and the depth of penetration of the sound by varying the field intensity[2)].

Thus, using the discussed procedures we can count on obtaining information concerning the function $\rho_s(\vec{r})$. The form of this function depends on the expression for the free energy, and consequently the choice of this expression can be checked by comparison with experiment. Specifically, measureement of the phase shift (or delay), depending on the position of the "mirror," makes it possible to determine $d\Psi/dx$, from which we can get also $\partial\Omega/\partial\Psi = (\hbar^2/m)d^2\Psi/dx^2$ [5] (we emphasize that no prior assumptions need be made here concerning the form of $\Omega(\mu, T, \Psi)$. By studying the reflection of second sound from a solid wall without a field (and in general by going beyond the limits of the applicability of geometrical optics), one can verify the complete system of equations for He-II near the λ point, investigate the course of ρ_s near a wall, the boundary of helium with vapor, etc.

We believe that the study of the propagation of second sound near the λ point in inhomogeneous helium should be an important stage in the study of superfluidity.

[1] V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. 34, 1230 (1958) [Sov. Phys.-JETP 7, 858 (1958)].
[2] Yu. G. Mamaladze, ibid. 52, 729 (1967) [25, 479 (1967)].
[3] V. A. Slyusarev and M. A. Strzhemechnyi, ibid. 58, 1757 (1970) [31, 941 (1970)].
[4] A. A. Sobyanin, ibid. 61, 433 (1971) [34, 229 (1972)].
[5] A. A. Sobyanin, ibid. 63, 1780 (1972) [35, No. 5 (1973)].
[6] K. R. Atkins and I. Rudnick, Progr. Low Temp. Phys., ed. C. R. Gorter, V. VI, P. 37, 1970.
[7] M. Kriss and I. Rudnick, J. Low Temp. Phys. 3, 339 (1970).
[8] R. P. Henkel, E. N. Smith, and I. D. Reppy, Phys. Rev. Lett. 23, 1276 (1969).
[9] S. A. Scott, E. Guyon, and I. Rudnick, J. Low Temp. Phys. 9, 389 (1972).
[10] L. V. Kiknadze, Yu. G. Mamaladze, and O. D. Cheishvili, Proc. 10-th Internat. Conf. on Low Temp. Physics (in Russian), VINITI, Vol. 1, 1967, p. 491.
[11] L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. 35, 408 (1958) [Sov. Phys.-JETP 8, 282 (1959)].
[12] I. M. Khalatnikov, ibid. 57, 489 (1969) [30, 268 (1970)].
[13] G. Winterling, F. S. Homes, and T. I. Greytak, Phys. Rev. Lett. 30, 427 (1973).

[2)] In problems with spherical and cylindrical symmetry of $\rho_s(\vec{r})$ it becomes possible to observe focusing of second-sound beams. However, only beams emitted at the impact distance $p > p_c$ are focused. When $p < p_c$ the beams fall on the "center" and p greatly exceeds the radius r_0 of the boundary of the layer of normal helium. Specifically, we have $p_c/r_0 = (5/3)^{1/2}(5/2)^{1/3} \approx 1.75$ for a charged filament and $p_c/r_0 = (7/3)^{1/4}(7/4)^{1/3} \approx 1.5$ for a charged sphere. Since usually $r_0 \gg \ell_E$, the use of focusing for the study of the course of ρ_s in a region where the correlation effects are appreciable is hardly possible.