CONCERNING THE STRUCTURE OF A CHARGED SURFACE OF LIQUID HELIUM

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The case of sufficiently strong fields, when the electron surface density is not small, is considered. It is shown that there exists a critical density, starting with which the homogeneous system is unstable. A method of observing the Wigner crystal structure is indicated.

It was shown in [1, 2] that localized electrons with approximate binding energy 10^{-3} eV (~ 10°K) can exist on the surface of helium. The appearance of bound states is due to the large barrier (~ 1 eV) to the passage of the electron into the helium and to the action of the image forces, which are small and of the order of the difference $\varepsilon_1 - \varepsilon_2 = 0.06$ between the dielectric constants in the liquid and gas phases. This question was investigated experimentally in [3-5]. The results of [3 - 5] indicate that stable localization of the electrons at approximately 1°K can be obtained by superimposing an additional "clamping" electric field. It was noted in [3] that, owing to the Coulomb character of the spectrum, the electrons are localized on the surface in the absence of a field only at very low temperatures.

We consider here the case of sufficiently strong fields (10^3 - 10^4 V/cm), when the surface density of the electrons is not small. It will be show that there exists a critical density, starting with which the homogeneous system is unstable. A periodic superstructure is then produced on the surface, with dimensions on the order of the capillary constant. In connection with the experiments of [5], we indicate a method of observing the Wigner crystal structure [6], the possible existence of which in the investigated problem was first pointed out in [7].

The experimental situation we have in mind coincides with that of [3] and is shown in the figure. The surface of the helium is at $z = 0$, and the electrons are clamped to it by a potential difference V applied to the metallic plate A.

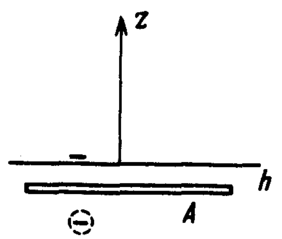
If there is no potential difference, then the Coulomb repulsion between the electrons prevails over the image forces at $z \geq (\varepsilon_1 - \varepsilon_2)r_s$. Therefore when $r_s < z_0/(\varepsilon_1 - \varepsilon_2)$, where r_s is the mean distance between electrons and $z_0 \sim 10^{-6}$ cm is the characteristic dimensions of the localized state [1, 2] the electrons become delocalized in the absence of a field. This corresponds exactly to densities $n_s \sim 10^8 - 10^9$ cm^{-2}. At lower densities and at finite temperatures the electrons "become ionized" and move away from the surface layer because of the infinite number of levels in the potential well of the image forces [3].

In the clamping field, the transverse-motion energy $\varepsilon_\perp$ and the characteristic dimension $\bar{z}$ of the state are estimated in the usual manner:

$$\varepsilon_\perp \sim \left(\frac{\hbar^2}{2m}\right)^{1/3}(eE)^{2/3} \sim Ry\left(\frac{a_0}{r_s}\right)^{4/3}, \quad \bar{z} \sim a_0^{1/3} r_s^{2/3}, \tag{1}$$

where $a_0 = 0.5\times10^{-8}$ cm is the Bohr radius and $E = 4\pi e n_s = 4\pi e r_s^{-2}$. It is seen from this estimate that $\bar{z}$ is small in comparison with the mean distance r_s between electrons. The electrons can therefore be regarded as lying in the plane. At low temperatures it can be assumed that the electrons form a "Wigner crystal" [6, 7]; since the kinetic energy is low, the ground state takes the form of a lattice of localized electrons. The kinetic energy is taken into account as a perturbation and contributes to the energy of the zero-point oscillations. The oscillator energy for a planar hexagonal lattice, calculated in [7], is equal to

$$U(r) = 2.7\frac{e^2}{r_s^3}r^2. \tag{2}$$



The characteristic oscillation frequency $\hbar\bar{\omega} \simeq 4.6\ Ry(a_0/r_s)^{3/2}$ corresponds to approximately 1°K already at $r_s \sim 0.5\times10^{-4}$ cm. The usual criteria with respect to "melting" of the lattice demonstrate by the same token the possibility of observing "Wigner crystallization" at the usual helium temperatures.

Measurement [5] of the cyclotron-resonance frequency at T = 1.2°K has shown that the latter depends only on the z-component of the magnetic field. By the same token, this demonstrates the two-dimensional character of the electron motion. We note that measurements of the field dependence of the resonance frequency can provide an unambiguous anser to the question whether the electrons make up a lattice. Indeed, the kinetic energy of the electrons in the magnetic field is given by $\hbar^2(\hat{p} - eA)^2/2m$. If $H \parallel z$ and $A_y = -Hx$, we get from this and from (2) that the resonant frequency for absorption of electromagnetic radiation by the oscillator takes the form $\omega_0 = \sqrt{\omega^2 + \omega_c^2}$, where ω_c is the usual cyclotron-resonance frequency. At $\tilde{\omega} \gg \omega_c$ the dependence on the field is quadratic. In the experiments of [5], $\omega_c \sim \tilde{\omega} \sim 1°K$. There are no plots of the resonance frequency against the field in [5]. The line width may be due also to the fact that the electron oscillations propagate over the lattice. The last question will be considered separately.

We now discuss the stability of the electron system on the surface of the helium. It is obvious that the Coulomb repulsion of the electrons tends to bend the surface. This is prevented by the surface tension of the helium.

The distribution of the electric field E is determined by the potential

$$\phi = e \sum_i \left(\frac{1}{r_i} - \frac{1}{r_i'} \right) - \mathcal{E} z, \tag{3}$$

where r_i and r_i' are the distances from the point $\vec{r}$ to the i-th lattice point and to its image, respectively, at $z = -h$ (see the figure). At distances z that are large in comparison with the lattice constant r_s, the z-component of the field E_z differs from zero:

$$e\bar{E}_z = e\mathcal{E}_z + 2\pi e^2 n_s \left(\frac{z}{|z|} - 1 \right).$$

The stability limit is obtained by solving the problem of small surface oscillations with wavelength $\lambda \gg r_s$. The displacement of the j-th lattice point is (u_j, η_j), where u_j is a vector in the plane $z = 0$. Expanding the expression for the electric field $E = -\nabla\phi$, where $\phi(z)$ is given by (3), in terms of the lattice deformations $u_j - u_i$ and $\eta_j - \eta_i$ ($u_{ix} = u_0 \exp(ikx)$, $\eta_i = \eta_0 \exp(ikx)$, we can easily calculate the forces acting on a given lattice point:

$$eE_z = -2\pi e^2 n_s^\circ + 2\pi e^2 n_s^\circ k \eta_0 (1 + 2\exp(-2kh)) + i\, 2\pi e^2 n_s^\circ k u_0 \exp(-2kh)$$

$$eE_x = -2\pi e^2 n_s^\circ k u_0 (1 - \exp(-2kh)) - i\, 4\pi e^2 n_s^\circ k \eta_0 \exp(-2kh).$$

The condition $E_x + ik\eta_0 E_z = 0$, which means that the force component tangent to the surface is equal to zero, yields a relation between u_0 and η_0:

$$u_0 = -i\eta_0 \frac{1 + 2\exp(-2kh)}{1 - \exp(-2kh)}.$$

The additional pressure due to the electric field is

$$\Delta P = -eE_z(n_s^\circ + \delta n_s) = -eE_z n_s^\circ \left(1 - \frac{\partial u_x}{\partial x}\right).$$

The dispersion of the surface oscillations can be expressed, after simple calculations in the form

$$\omega^2 = \frac{k}{\rho} \operatorname{th}(kh) \{ \rho g + \alpha k^2 - 2\pi e^2 n_s^{0\,2} k (1 + 2\exp(-2kh)) (\operatorname{cth}(kh) + 1) \}. \tag{4}$$

In the limiting case $kh \gg 1$, expression (4) coincides with the spectrum of the oscillations on the surface of a liquid conductor (see [8]). The stability limit in this case is $e^2 n_s^2 < \sqrt{\rho g \alpha}/2\pi$, which yields for helium $n_{cr} \sim 2.2\times10^9$ cm^{-2} and a field $E_{cr} \sim 4000$ V/cm. At $kh \ll 1$ we have $\omega^2 = k^2 h/\rho[g\rho - (2\pi e^2 n_s/h)]$.

The instability in (4) corresponds to $k_0 \simeq a^{-1}$ ($a \sim 0.05$ is the capillary constant of H). At $n_s > n_{cr}$ there develops on the helium surface a periodic structure, initially in the form of bands of wavelength $\lambda \sim 2\pi a \simeq 0.3$ cm. In [3], where a photograph of a charged helium surface is shown, the conditions are already close to those needed to observe the predicted effect, viz., $n_s \sim 10^9$ cm^{-2} and $E \simeq 2000$ V/cm. The instabilities described in [5], which set in when the surface is being charged, are apparently connected with the same phenomenon. We note also that the periodic structure remains stable in the entire interval $n_s > n_{cr}$, and because of its long-wave nature it does not disturb the crystallization of the electrons into a Wigner lattice.

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STRATIFICATION OF CURRENT OR FIELD IN SYSTEMS WITH POSITIVE DIFFERENTIAL RESISTANCE

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It is shown that instability of the current or field with respect to fluctuations with $k \neq 0$ can arise in systems whose properties depend on two parameters that have different spatial dispersion, if the differential resistance is positive, i.e., even if the current-voltage characteristic of the system is single-valued.

Homogeneous distribution of the current or field in a system with negative differential resistance is unstable [1, 2]. We show in this communication that a spatial instability, viz., a stratification of the current (field), can arise in a system with positive differential resistance. The latter situation is realized, under definite conditions, in systems whose current-voltage characteristic depends on two (or more) parameters having different spatial dispersion, $I = f(x, y, V)$.

Indeed, let the parameters x and y satisfy equations of the type[1)]

$$\tau_1 \frac{\partial x}{\partial t} = L^2 \vec{\Delta} x - Q(x, y, V), \quad \tau_2 \frac{\partial y}{\partial t} = \ell^2 \vec{\Delta} y - q(x, y, V). \tag{1}$$

The differential resistance of such a system is

$$\sigma_g = R_g^{-1} = f'_v + f'_x \frac{(Q'_y q'_v - q'_y Q'_v)}{(Q'_x q'_y - Q'_y q'_x)} + f'_y \frac{(Q'_x q'_x - q'_v Q'_x)}{(Q'_x q'_y - Q'_y q'_x)}, \tag{2}$$

where the prime denotes the corresponding partial derivatives ($Q'_x = \partial Q/\partial x$). Linearizing the equations in (1) at V = const[2)] with respect to perturbation of the type $\Delta\alpha = \Delta\alpha_0 \exp(i\omega t - i\vec{k}\cdot\vec{r})$, we obtain the dispersion equation

$$\tau_1\tau_2(i\omega)^2 - i\omega[\tau_1(q'_y + \ell^2k^2) + \tau_2(Q'_x + L^2k^2)] - q'_x Q'_y + (Q'_x + L^2k^2) \times \times (q'_y + \ell^2k^2) = 0, \tag{3}$$

from which it follows that the instability sets in ($\mathrm{Im}\,\omega < 0$) if one of the following conditions is satisfied:

$$\tau_1(q'_y + \ell^2k^2) + \tau_2(Q'_x + L^2k^2) < 0, \tag{4}$$

$$L^2\ell^2k^4 + \ell^2k^2Q'_x + L^2k^2q'_y + Q'_x q'_y - q'_x Q'_y < 0. \tag{5}$$

It follows from inequality (5) that the homogeneous current distribution is certainly stable if