face is being charged, are apparently connected with the same phenomenon. We note also that the periodic structure remains stable in the entire interval $n_s > n_{cr}$, and because of its long-wave nature it does not disturb the crystallization of the electrons into a Wigner lattice.

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STRATIFICATION OF CURRENT OR FIELD IN SYSTEMS WITH POSITIVE DIFFERENTIAL RESISTANCE

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It is shown that instability of the current or field with respect to fluctuations with $k \neq 0$ can arise in systems whose properties depend on two parameters that have different spatial dispersion, if the differential resistance is positive, i.e., even if the current-voltage characteristic of the system is single-valued.

Homogeneous distribution of the current or field in a system with negative differential resistance is unstable [1, 2]. We show in this communication that a spatial instability, viz., a stratification of the current (field), can arise in a system with positive differential resistance. The latter situation is realized, under definite conditions, in systems whose current-voltage characteristic depends on two (or more) parameters having different spatial dispersion, $I = f(x, y, V)$.

Indeed, let the parameters x and y satisfy equations of the type[1)]

$$\tau_1 \frac{\partial x}{\partial t} = L^2 \vec{\Delta} x - Q(x, y, V), \quad \tau_2 \frac{\partial y}{\partial t} = \ell^2 \vec{\Delta} y - q(x, y, V). \tag{1}$$

The differential resistance of such a system is

$$\sigma_g = R_g^{-1} = f'_V + f'_x \frac{(Q'_y q'_V - q'_y Q'_V)}{(Q'_x q'_y - Q'_y q'_x)} + f'_y \frac{(Q'_x q'_x - q'_V Q'_x)}{(Q'_x q'_y - Q'_y q'_x)}, \tag{2}$$

where the prime denotes the corresponding partial derivatives ($Q'_x = \partial Q/\partial x$). Linearizing the equations in (1) at V = const[2)] with respect to perturbation of the type $\Delta\alpha = \Delta\alpha_0 \exp(i\omega t - i\vec{k}\cdot\vec{r})$, we obtain the dispersion equation

$$\tau_1 \tau_2 (i\omega)^2 - i\omega[\tau_1(q'_y + \ell^2 k^2) + \tau_2(Q'_x + L^2 k^2)] - q'_x Q'_y + (Q'_x + L^2 k^2) \times \\ \times (q'_y + \ell^2 k^2) = 0, \tag{3}$$

from which it follows that the instability sets in ($\mathrm{Im}\,\omega < 0$) if one of the following conditions is satisfied:

$$\tau_1 (q'_y + \ell^2 k^2) + \tau_2 (Q'_x + L^2 k^2) < 0, \tag{4}$$

$$L^2 \ell^2 k^4 + \ell^2 k^2 Q'_x + L^2 k^2 q'_y + Q'_x q'_y - q'_x Q'_y < 0. \tag{5}$$

It follows from inequality (5) that the homogeneous current distribution is certainly stable if

$$Q_x' q_y' > q_x' Q_y'. \quad (6)$$

It is known on the other hand [1] that a sufficient condition for such an instability is the presence of a negative differential resistance. This means that the last two terms of the inequality (5), while not exactly coinciding with (2), determine the sign of the differential resistance. This takes place in all known cases [1 - 4], including the example presented below. This is natural, for the opposite would correspond to the following two situations: If $R_g < 0$, and inequality (6) is not satisfied, this means that in spite of the presence of negative differential resistance the homogeneous current distribution is stable. This contradicts the conclusions of [1 - 4]. If on the other hand $R_g > 0$ in spite of the satisfaction of condition (6), then this corresponds to the situation of interest to us, wher the homogeneous current distribution is unstable at a positive differential resistance of the system. The latter takes place when the numerator of (2) goes through zero simultaneously with the denominator. We cannot cite, however, a concrete example of a system in shich this situation is realized.

We discuss now the conditions under which the inequalities (4) and (5) are satisfied as $k \to 0$. In that case, when $q_x' Q_y' < 0$, condition (4) can be satisfied also if $R_g > 0$. Such a situation is realized, for example, when $\tau_1 >> \tau_2$, and q_y' becomes negative at a certain voltage on the sample, a case corresponding to the presence of "hidden" negative differential resistance with respect to one of the parameters, y. Satisfaction of the inequality (4) means that the dynamic resistance of the system becomes negative at a certain frequency. An instability of this type in homogeneous semiconductors was considered in [4]. (We note that in our case such an instability develops with respect to long-wave fluctuations, unlike the main instability of [4]).

The main result of this paper is the conclusion that if either the static or the dynamic differential resistance is positive, i.e., when the inequalities (4) and (5) are not satisfied at $k = 0$, an aperiodic instability can set in at $k > 0$. Indeed, as follows from inequality (5), the stability of the homogeneous distribution is disturbed when

$$q_y' = -\left[\left(\frac{\ell}{L}\right)^2 Q_x' + 2\left(\frac{\ell}{L}\right)(Q_x' q_y' - q_x' Q_y')^{1/2}\right] \text{ at } k_{cr} = (\ell L)^{1/2}(Q_x' q_y' - q_x' Q_y')^{1/4}, \quad (7)$$

from which it is seen directly that when $L >> \ell$ the condition (7) with $q_y' < 0$ and $Q_x' > 0$ can be satisfied when the conditions (4) and (6) are not satisfied. An aperiodic instability then sets in at a critical voltage determined by (7), only with respect to a definite $k = k_{cr}$.

The physical meaning of this result is as follows. Owing to the different spatial dispersion of the parameters, the more "rigid" of them cannot follow the fluctuations of the other (y) with $k \simeq (\ell L)^{-1/2}$, i.e., a "spatial decoupling" of the parameters takes place at relatively large k. It follows therefore that the present of negative resistance with respect to the "soft" parameter should lead to a stratification of the current (field). (Fluctuations with very large k attenuate because of the large diffusion fluxes they produce.)

The conditions obtained for the stratification of the current at positive differential resistance can be satisfied both in homogeneous semiconductors a d in semiconducting structures. The simplest and at the same time important example in which this instability is realized is Joule heating of a semiconducting p-n-p structure. The parameters that determine the current of such a structure are the voltage on one of the p-n junctions, V_1, and the temperature T. These parameters satisfy, respectively, the following current continuity conditions in the n-region, averaged along the z axis perpendicular to the plane of the p-n junctions [2]:

$$C\frac{\partial V_1}{\partial t} = W_0 \vec{\Delta}_\perp V_1 - I_1(V_1, T, V) + I_2(V_1, T, V) \quad (8)$$

and the averaged heat-conduction equation [5]

$$\ell_z \rho c \frac{\partial T}{\partial t} = \ell_z \kappa \vec{\Delta}_\perp T + I_1 V_1 + I_2(V - V_1) - \frac{T - T_0}{\tau}\ell_z \rho c, \quad (9)$$

where

$$I_1 = I_{s1}(T)\left(\exp\frac{eV_1}{kT} - 1\right) + \alpha_2 I_{s2}(T),$$

$$I_2 = \alpha_1 I_{s1}(T)\left(\exp\frac{eV_1}{kT} - 1\right) + I_{s2}(T) + \sigma_R(V - V_1)$$

are the current densities through the first and second p-n junctions, $I_{s1}(T)$, $I_{s2}(T)$, α_1, and α_2 are the saturation currents of these junctions and the current gains corresponding to them, W and σ are the thickness and conductivity of the n-region, σ_k is the specific conductivity of the reverse-biased p-n junction [2], C is the specific capacitance of the p-n junctions, ρ, c, and κ are respectively the density, specific heat, and thermal conductivity of the material, ℓ_z is the thickness of the structure, and τ is the temperature relaxation time. As seen from (8) and (9), the characteristic length of potential variation is $L = \sqrt{kT_0\sigma W/eI_s(T_0)}$ [2], and that of the temperature variation is $\ell = \sqrt{\kappa\tau/\rho c}$ [5]. For real parameters we have $L \gg \ell$, and it is easy to verify that all the conditions under which the current becomes stratified at a positive differential resistance of the structure are satisfied. This effect is observed in experimental investigations of the thermal breakdown of such structures [6, 7].

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1) Conditions of this type are satisfied, for example, by the equations for the effective temperature of hot electrons [1] and phonons [3] and for the distribution of the potentials across p-n junctions [2] in systems in which pinching of the current takes place

2) Inhomogeneous perturbations do not change the total current in the circuit, and hence also the voltage across the sample [1, 2].

SURFACE OSCILLATIONS OF A DROP OF FERMI LIQUID

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The collective properties of a drop of Fermi liquid are investigated on the basis of the self-consistency conditions Formulas are obtained for the calculation of the surface-oscillation spectrum characteristics. It is shown tat at large L the spectrum of the surface oscillations of a drop of Fermi liquid is hydrodynamic.

In an earlier paper [1], the author attempted to determine the conditions under which a Fermi system of finite (but sufficiently large) dimensions behaves like a liquid drop, namely, its radius increases like $R = r_0N^{1/3}$ and consequently even when a small number k of particles is added the system density changes mainly at the edge, where this change is of the order of $\delta\rho(r \simeq R) \sim (\partial\rho/\partial R)\delta R \sim k/N^{2/3}$ (for a gas of particles contained in a box of the same dimensions, the change is $\delta\rho(r \simeq R) \sim k/N$). Such a behavior of $\delta\rho$ for a drop is uniquely connected with the existence of a spectrum of low-lying surface collective excitations, the characteristics of which are expressed in standard fashion in terms of the parameters C_L and B_L of the collective