$$I_2 = \alpha_1 I_{s1}(T)\left(\exp\frac{eV_1}{kT} - 1\right) + I_{s2}(T) + \sigma_R(V - V_1)$$

are the current densities through the first and second p-n junctions, $I_{s1}(T)$, $I_{s2}(T)$, α_1, and α_2 are the saturation currents of these junctions and the current gains corresponding to them, W and σ are the thickness and conductivity of the n-region, σ_k is the specific conductivity of the reverse-biased p-n junction [2], C is the specific capacitance of the p-n junctions, ρ, c, and κ are respectively the density, specific heat, and thermal conductivity of the material, ℓ_z is the thickness of the structure, and τ is the temperature relaxation time. As seen from (8) and (9), the characteristic length of potential variation is $L = \sqrt{kT_0\sigma W/eI_s(T_0)}$ [2], and that of the temperature variation is $\ell = \sqrt{\kappa\tau/\rho c}$ [5]. For real parameters we have $L \gg \ell$, and it is easy to verify that all the conditions under which the current becomes stratified at a positive differential resistance of the structure are satisfied. This effect is observed in experimental investigations of the thermal breakdown of such structures [6, 7].

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[1)] Conditions of this type are satisfied, for example, by the equations for the effective temperature of hot electrons [1] and phonons [3] and for the distribution of the potentials across p-n junctions [2] in systems in which pinching of the current takes place

[2)] Inhomogeneous perturbations do not change the total current in the circuit, and hence also the voltage across the sample [1, 2].

SURFACE OSCILLATIONS OF A DROP OF FERMI LIQUID

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The collective properties of a drop of Fermi liquid are investigated on the basis of the self-consistency conditions Formulas are obtained for the calculation of the surface-oscillation spectrum characteristics. It is shown tat at large L the spectrum of the surface oscillations of a drop of Fermi liquid is hydrodynamic.

In an earlier paper [1], the author attempted to determine the conditions under which a Fermi system of finite (but sufficiently large) dimensions behaves like a liquid drop, namely, its radius increases like $R = r_0N^{1/3}$ and consequently even when a small number k of particles is added the system density changes mainly at the edge, where this change is of the order of $\delta\rho(r \simeq R) \sim (\partial\rho/\partial R)\delta R \sim k/N^{2/3}$ (for a gas of particles contained in a box of the same dimensions, the change is $\delta\rho(r \simeq R) \sim k/N$). Such a behavior of $\delta\rho$ for a drop is uniquely connected with the existence of a spectrum of low-lying surface collective excitations, the characteristics of which are expressed in standard fashion in terms of the parameters C_L and B_L of the collective

Hamiltonian

$$H(\alpha) = \frac{1}{2}\Sigma C_L |\alpha_{L\mu}|^2 + \frac{1}{2}\Sigma B_L |\dot{\alpha}_{L\mu}|^2. \quad (1)$$

In [1] we considered a simple model, in which it was assumed that the local interaction Γ^ω between the quasiparticles differs from zero only on the system surface, where it coincides with the vacuum value Γ^{vac}. It was found that the system acquires the properties of a liquid when the dimensionless parameter $f = \Gamma^{vac} p_F M^*/\pi^2$ (p_F is the Fermi momentum) becomes close to the critical value f_c, which does not depend on the system dimensions and at which the frequency of the monopole oscillations vanishes.

It will be shown in this paper that a real system retains many features of the model of [1]. We begin with a consideration of the self-consistency condition for the quasiparticle Hamiltonian $H_p(r, p) = (p^2/2m^*) + V(r)$, the Green's function of the quasiparticle $G^q(r,p,\varepsilon) = a(r)(\varepsilon - H_q)^{-1}$, and the local interaction $\Gamma^\omega(r)$. This condition is easiest to derive by stipulating that the frequency ω_1 of the dipole oscillation be equal to zero, since such an oscillation is none other than a shift of the center of gravity of the system. This means that the standard equation of the theory of finite Fermi systems [2], which determines the spectrum of collective excitations of multipolarity L (se confine ourseves here to the zeroth harmonic of the local interaction Γ^ω)

$$g_L(r, \omega_S) = \Gamma^\omega(r) \int A_L(r, r'\omega_S)\, g_L(r', \omega_S)\, d\tau' \quad (2)$$

where $d\tau = r^2 dr$ and

$$A_L(r, r', \omega) = \frac{1}{4\pi} \iint A(r, r'\omega) P_L(nn')\, dn\, dn' \quad (2')$$

$$A(r, r', \omega) = \int G^q(r, r', \epsilon + \frac{\omega}{2})\, G^q(r', r, \epsilon - \frac{\omega}{2})\, d\epsilon/2\pi i \quad (2'')$$

has a solution at $\omega_1 = 0$.

As is well known, the amplitude $g_L(r)$ is proportional to the change of the self-consistent field $\Sigma(r)$ on going from the ground state to the collective excitation of the system. Similarly, the amplitude $g_L^+(r)$, which satisfies an equation conjugate to (2), is proportional to the change in the density $\rho^q(r) = JG^q(r, p, \varepsilon)d\varepsilon/2\pi i$ on going to the ground to the collective state. It follows from the foregoing that $g_1(r) \sim \partial\Sigma(r, \mu)/\partial r$, where

$$\frac{\partial \Sigma(r, \mu)}{\partial r} = \Gamma^\omega(r) \int A_1(r, r'\omega = 0) \frac{\partial \Sigma(r', \mu)}{\partial r'} d\tau' \quad (3)$$

since $g_1(r) \sim \Sigma(\vec{r} + \delta\vec{R}) - \Sigma(\vec{r}) \sim (\partial\Sigma/\partial r)\delta R$, where δR is the shift of the center of gravity of the system, and $\Sigma(r, \mu) = (M^*/M)V(r) - \mu(M^*/M - 1)$ [2]. Similarly, $g_1^+ \sim \partial\rho''/\partial r$, and we can rewrite (3) in the form

$$\frac{\partial \rho^q(r)}{\partial r} = \int A_1(r, r', \omega = 0)\Gamma^\omega(r') \frac{\partial \rho^q(r')}{\partial r'} d\tau' \quad (4)$$

(the self-consistency condition was derived in a different form in [3]; the author thanks Dr. Mayer for pointing out this paper). A rough estimate of the value of $f^{vac} = \Gamma^{vac} p_F M/\pi^2$ at which a nontrivial solution of (4) first appears can be obtained by replacing Γ^ω in (4) by ^{vac}, since the difference between Γ^ω and Γ^{vac} becomes immaterial in the vicinity of the critical point, where the surface becomes smeared out. If we use in (4) the quasiclassical formula $A(\vec{r}_1, \vec{r}_2) = (p_F M/\pi^2)\delta(\vec{r}_1 - \vec{r}_2)$, then we get $f_c^{vac} = -1$. We note that a neutron liquid satisfies this rough criterion, since the value $f_m^{vac} = -1.4$ was calculated by Yu. V. Gaponov for free neutrons scattered at energies $\varepsilon \sim 40$ MeV [2].

It is easy to verify with the aid of (4) that there exists an entire spectrum of low-lying surface oscillations. Indeed, the propagator $A(\vec{r}_1, \vec{r}_2)$ defined by the integral (2'') has a sharp δ-like peak at the point $\vec{r}_1 = \vec{r}_2$ with a width $|\vec{r}_1 - \vec{r}_2| \sim r_0$. For the considered case of surface

waves, this peak makes the decisive contribution to (2), and therefore at $L < N^{1/3}$ the Legendre polynomial $P_L(x)$ can be taken outside the integral sign in (2) at the point $x = 1$, in first-order approximation, as a result of which A_L no longer depends on L. Then, obviously $\omega_L = \omega_1 = 0$ and $g_L = g_1 \sim \partial\Sigma/\partial r$, i.e., the surface-oscillation spectrum becomes degenerate. In the higher approximations this degeneracy is lifted for the following reasons: 1) the presence of long-range components in $A(r_1, r_2)$ gives rise in $g_L(r)$ to volume corrections $g_L^{in}(r)$, the amplitude of which decreases with increasing L: $g_L^{in} \sim N^{-1/3}L^{-1}g_1(k)$; 2) the contribution of the local part of $A(r_1, r_2)$ to the integral (2') also decreases with increasing L. At $L > 1$, the surface-oscillation frequencies can be obtained from the relation

$$\omega^2 = \iint \frac{\partial \Sigma(r_1)}{\partial r_1}\left(A_1(r_1, r_2, \omega = 0) - A_L(r_1, r_2\, \omega = 0)\right)\frac{\partial \Sigma}{\partial r_2} dr_1 dr_2 / \iint \frac{\partial \Sigma(r_1)}{\partial r_1} \times$$
$$\times \left(\frac{dA_L(r_1 r_2)}{d\omega^2}\right)_0 \frac{\partial \Sigma}{\partial r_2} dr_1 dr_2 \qquad (5)$$

which follows from (2) and (3). In the considered case of $L > 1$, the numerator of (5), which determines the surface rigidity C_L, can be simplicied by substituting in (2') the expansion $P_L(x) = 1 - L(L + 1)(1 - x)/2$, from which it follows that

$$A_1(r_1, r_2) - A_L(r_1, r_2) = \frac{L(L+1)}{8\pi} \iint A(r_1, r_2)(1 - n_1 n_2)\, dn_1 dn_2. \qquad (6)$$

Comparing now (5) with the hydrodynamic rigidity $C_L \sim \sigma R^2(L - 1)(L + 2)$, where σ is the surface-tension coefficie-t, we can obtain the following formula for σ (in the approximation where the radius of the interaction forces is zero)

$$\sigma = \frac{1}{8\pi} \iint \frac{\partial \Sigma(r_1)}{\partial r_1} A(r_1, r_2, \omega = 0)\,(1 - n_1 n_2) \frac{\partial \Sigma(r_2)}{\partial r_2} d^3 r_1 d^3 r_2. \qquad (7)$$

A rough calculation of this integral in the square-well model yields for the nucleus a value $4\pi R^2\sigma \simeq 25A^{2/3}$ MeV, which differs from the experimental value by only 30%.

The calculation of the right-hand side is somewhat more complicated. After rather lengthy manipulations, we obtain $(dA_L/d\omega^2) \sim L^{-1}$, from which it follows that $\omega_L^2 \sim L^3/N$, i.e., the spectrum of surface oscillations with $L > 1$ is hydrodynamic. Let us turn, in conclusion, again to small L, or more accurately to the case $L = 0$. It is easily seen that unlike in ordinary hydrodynamics, the surface oscillations includes also a monopole oscillation ($L = 0$). Its frequency is $\omega_0 \sim \varepsilon_F N^{-1/3}$, and its amplitude $g_0(r, \omega)$ (see (2)) has a sharp maximum on the surface of the system. Although no 0^+ level with such properties has been observed as yet (a possible candidate is the first level in ^{16}O at 6.06 MeV), its influence on the behavior of the system is most appreciable, for no matter where the particles are added, they excite this collective level strongly, as a result of which the greatest change of density occurs on the edge of the system and its volume increases in proportion to the number of particles. To see this, we write down the usual equation for the change of density $\delta\rho$ (more accurately, of the spherically-symmetrical component) following the addition of several particles [2]

$$\delta\rho(r) = \delta_0\rho(r) + \int A_0(r, r', \omega = 0)\Gamma^{\omega}(r')\delta\rho(r')\, dr', \qquad (8)$$

where $\delta_0\rho(r) = \Sigma\phi^2_{\lambda_i}(r)$.

Bearing relation (4) in mind, it is convenient to separate from A_0 the surface term

$$A_0(r_1, r_2) = \kappa_0 \frac{\partial \rho^q(r_1)}{\partial r_1} \frac{\partial \rho^q(r_2)}{\partial r_2} + \mathfrak{A}_0(r_1, r_2), \qquad (9)$$

where κ_0 is defined by the requirement

$$\int \frac{\partial \rho^q(r_1)}{\partial r_1} \mathfrak{A}_0(r_1, r_2) \frac{\partial \rho^q(r_2)}{\partial r_2} dr_1 dr_2 = 0. \qquad (10)$$

As a result of such a subdivision of A_0, we can separate in $\delta\rho$ accurately the surface contribution and write down the solution of (8) in the form

$$\delta\rho(r) = \delta_1\rho(r) + \nu_0\delta_2\rho(r), \tag{11}$$

where $\delta_1\rho$ satisfies the equation

$$\delta_1\rho(r) = \delta_0\rho(r) + \int \mathcal{A}_0(r, r')\Gamma^{\omega}(r')\delta_1\rho(r')\,dr', \tag{12}$$

the solution of which is insensitive to details of the behavior of $\Gamma^{\omega}(r)$ in the transition layer. The function $\delta_2\rho(r)$, defined by the equation

$$\delta_2\rho(r) = \frac{\partial\rho^{q}(r)}{\partial r} + \int \mathcal{A}_0(r, r')\Gamma^{\omega}(r')\delta_2\rho(r')\,dr' \tag{13}$$

has a sharp maximum on the surface and is small on the inside. The constant ν_0 is defined by the relation

$$\nu_0 = \kappa_0 \int \frac{\partial\Sigma(r)}{\partial r}\delta_1\rho(r)\,dr \Big/ \left(1 - \kappa_0 \int \frac{\partial\Sigma(r)}{\partial r}\delta_2\rho(r)\,dr\right). \tag{14}$$

The denominator of this expression is small so that we can estimate ν_0 at $\nu_0 \sim N^{-1}/N^{-1/3} \sim N^{-2/3}$, whence $\delta\rho(r \simeq R) \sim k/N^{2/3}$. This means that a real nucleus behaves not like a gas of interacting quasiparticles, but like a liquid.

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GAMOW-TELLER RESONANCE AND WIGNER MULTIPLET SCHEME

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Quasiclassical solutions for the energies and matrix elements of the Gamow-Teller resonance are obtained within the framework of the theory of finite Fermi systems. It is shown that in medium and heavy nuclei this resonance is described in the Wigner supermultiplet scheme, and belongs together with the analog state to the supermultiplet (T_0, 0, 0).

The investigation of the isobaric I^+ states in microscopic nuclear theory has shown [1, 2] that there exists a distinct tendency towards collectivization of the $p\bar{n}$ branch of the states of this type, so that a preferred collective isobaric I^+ state, the hypothetical Gamow-Teller (GT) resonance, should exist. This state lies in the region of the analog state and has apparently been observed in the region of light [3] and medium [1] nuclei.

In the theory of finite Fermi systems [5], the characteristics of the collective isobaric state can be obtained in the quasiclassical approximation by the method of [6]. In terms of the parameters Δe_F (the energy width of the excess-neutron layer), δe_F (the relative displacement of the Fermi p and n surfaces) and ε (the average spin-orbit energy of the last shell), and in the approximation $\Delta e_F > 2\varepsilon$, the GT-resonance energy is

$$\omega = \delta e_F + \Delta e_F + (a + b)g_0'\Delta e_F + a\varepsilon;\quad a = \frac{\varepsilon}{g\Delta\epsilon_F} + c;\quad g = g_0'\frac{a+b}{1 + b g_0'} \tag{1}$$

The energy ω is reckoned from the ground state of the even-even nucleus A(N, Z), and the GT resonance is observed in the nucleus A(N - 1, Z + 1). Assuming the system of functions to be