As a result of such a subdivision of $A_0$, we can separate in $\delta\rho$ accurately the surface contribution and write down the solution of (8) in the form

$$\delta\rho(r) = \delta_1\rho(r) + \nu_0\delta_2\rho(r), \tag{11}$$

where $\delta_1\rho$ satisfies the equation

$$\delta_1\rho(r) = \delta_0\rho(r) + \int \mathcal{A}_0(r,r')\Gamma^{\omega}(r')\delta_1\rho(r')\,dr', \tag{12}$$

the solution of which is insensitive to details of the behavior of $\Gamma^{\omega}(r)$ in the transition layer. The function $\delta_2\rho(r)$, defined by the equation

$$\delta_2\rho(r) = \frac{\partial\rho^q(r)}{\partial r} + \int \mathcal{A}_0(r,r')\Gamma^{\omega}(r')\delta_2\rho(r')\,dr' \tag{13}$$

has a sharp maximum on the surface and is small on the inside. The constant $\nu_0$ is defined by the relation

$$\nu_0 = \kappa_0\int\frac{\partial\Sigma(r)}{\partial r}\delta_1\rho(r)\,dr \Big/ \left(1 - \kappa_0\int\frac{\partial\Sigma(r)}{\partial r}\delta_2\rho(r)\,dr\right). \tag{14}$$

The denominator of this expression is small so that we can estimate $\nu_0$ at $\nu_0 \sim N^{-1}/N^{-1/3} \sim N^{-2/3}$, whence $\delta\rho(r \simeq R) \sim k/N^{2/3}$. This means that a real nucleus behaves not like a gas of interacting quasiparticles, but like a liquid.

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## GAMOW-TELLER RESONANCE AND WIGNER MULTIPLET SCHEME

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Quasiclassical solutions for the energies and matrix elements of the Gamow-Teller resonance are obtained within the framework of the theory of finite Fermi systems. It is shown that in medium and heavy nuclei this resonance is described in the Wigner supermultiplet scheme, and belongs together with the analog state to the supermultiplet $(T_0, 0, 0)$.

The investigation of the isobaric $I^+$ states in microscopic nuclear theory has shown [1, 2] that there exists a distinct tendency towards collectivization of the $p\bar{n}$ branch of the states of this type, so that a preferred collective isobaric $I^+$ state, the hypothetical Gamow-Teller (GT) resonance, should exist. This state lies in the region of the analog state and has apparently been observed in the region of light [3] and medium [1] nuclei.

In the theory of finite Fermi systems [5], the characteristics of the collective isobaric state can be obtained in the quasiclassical approximation by the method of [6]. In terms of the parameters $\Delta e_F$ (the energy width of the excess-neutron layer), $\delta e_F$ (the relative displacement of the Fermi p and n surfaces) and $\epsilon$ (the average spin-orbit energy of the last shell), and in the approximation $\Delta e_F > 2\epsilon$, the GT-resonance energy is

$$\omega = \delta e_F + \Delta e_F + (a+b)g_0'\Delta e_F + a\epsilon;\quad a = \frac{\epsilon}{g\Delta\epsilon_F} + c;\quad g = g_0'\frac{a+b}{1+bg_0'} \tag{1}$$

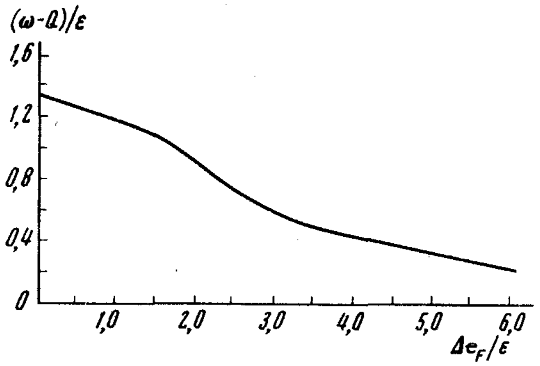
The energy $\omega$ is reckoned from the ground state of the even-even nucleus A(N, Z), and the GT resonance is observed in the nucleus A(N - 1, Z + 1). Assuming the system of functions to be complete, a = 1/3, b = 2/3, and c ≈ 0.3. The matrix element of the $\beta^+$ decay of the GT resonance in the $0^+$ ground state of the nucleus A(N, Z) is

$$M_{GT}^2 = e_q^2(N - Z)\left(1 + \frac{1}{3}\,\frac{\epsilon}{\Delta e_F}\right) \qquad e_q \approx 0,9. \tag{2}$$

Formulas (1) and (2) describe well the exact solutions [2] and explain the qualitative features of the GT resonance. Principal among them are the mutual approach of the GT and analog resonance energies with increasing $\Delta e_F$ ($\sim$N - Z), as illustrated in the figure, and the near-equality of the β-decay matrix elements for these states, $M_{GT} \approx M_F$.

These features are well interpreted in the phenomenological scheme of the Wigner supermultiplets [7]. Indeed, the value of the GT resonance β-decay matrix element can be explained if the GT resonance has an isospin $T_0 - 1$ (T is the isospin of the ground state of the A(N, Z) nucleus), since it follows from the isovector character of the GT vertex that $|\Delta T| = 1$ and

$$\frac{M_{GT}^2}{N - Z} = (2T_0 - 1)/(2T_0 + 1) \text{ for the } T_0 - 1 \to T_0 \text{ transitions.} \tag{3}$$




Relative position of the Gamow-Teller (ω) and analog (Q) resonances

The approximate degeneracy of the analog resonance which is contained together with the ground state of the A(N, Z) nucleus in the isomultiplet $(T, S^\pi) = (T_0, 0^+)$, and of the GT resonance belonging to the isomultiplet $(T_0 - 1, I^+)$, allows us to asume further that both isomultiplets are contained in a single Wigner supermultiplet that is split by the effective spin-orbit interaction. With increasing $e_F$, the relative role of the spin-orbit interaction, as can be seen from the figure, is diminished. The simplest supermultiplet unifying the analog and GT resonances is $(T_0, 0, 0)$. This conclusion is practically unique, since other multiplets include additional degenerate states whose interpretation is not clear.

Since other $I^+$ states of different nature with isospin $T_0 - 1$ should have in principle different supermultiplet symmetries, the coupling between the GT resonance and the surrounding states of the quasidiscrete spectrum should be weakened. The GT resonance should be observed in the form of a broad resonance (to the extent that the supermultiplet symmetry is broken by the spin-orbit and Coulomb interactions) against the background of compound states. The quasiclassical solution (1) leads us to expect that the symmetry is restored with increasing N - Z. A similar fact is observed in the reduction of the nuclear masses using the Wigner supermultiplet scheme [8]. The agreement was improved with increasing N - Z and A. It is also of interest to note that for even-even nuclei the agreement was obtained when they were interpreted as states of the supermultiplet $(T_0, 0, 0)$, which agrees with our conclusion.

We have thus observed an interesting connection between the problem of the GT resonance and the Wigner symmetry. Therefore experimental searches for GT resonances in medium and heavy nuclei are particularly desirable.

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## ELECTROMAGNETIC CONTRIBUTIONS TO TOTAL HADRON CROSS SECTIONS AT HIGH ENERGIES

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A large electromagnetic correction ($\sim\alpha \ln^2(s/s_0)$) can appear in processes with large multiplicity. This correction may explain the observed growth of the cross sections in ISR experiments.

1. In hadron collisions, an increase in energy leads to a rapid growth of the multiplicity, $\langle n_{ch}\rangle \gtrsim a \ln s + b$. Prompt production of a large number of charged particles is accompanied by strong electromagnetic radiation.

It is know that the relative momenta of the produced hadrons are not small on the average. The electromagnetic radiation of individual hadrons is therefore incoherent in the mean. This can be verified, for example, by considering the fluctuations of the squared change of a charge moving to the right in the c.m.s. In sum, the cross section of the accompanying radiation increases in the principal approximation in proportion to $\langle n_{ch}\rangle$. A simple calculation by means of the usual formulas (see, e.g., [1]) yields [2]

$$d\sigma_\gamma = \frac{\alpha}{2\pi}\frac{dk_\perp^2}{k_\perp^2}\frac{d\omega}{\omega}\langle n_{ch}\rangle \sigma_{tot}. \tag{1}$$

Here $\omega$ and $k_\perp$ are the frequency and transverse momentum of the photon.

Allowance for the correlations, if their radius is finite, does not change the principal conclusion that $d\sigma_\gamma$ is proportional to $\langle n_{ch}\rangle$. Thus, if the hadrons are produced in clusters (fireballs) with mean-squared charge $\langle Q^2\rangle$, and if the number of clusters is N, then the quantity $\langle n_{ch}\rangle$ in (1) is replaced by $N\langle Q^2\rangle \sim \langle n_{ch}\rangle$.

2. Integration of the cross section (1) leads to the usual expression containing $\langle \ln^2 q_i q_j\rangle$ ($q_i$ and $q_j$ are the momenta of the produced hadrons). When calculating the correction to the total cross section, it is necessary to add here the compensating contributions due to exchange of virtual photons. The assumption that the amplitudes decrease rapidly on going off the mass shell and with increasing $|t|$ lead to a complete cancellation of the doubly-logarithmic terms [3]. It is natural to assume, however, that at least one logarithm remains in the sum. One of the causes of its preservation may be (in the language of [3]) the numerical difference between the electromagnetic and strong radii of the hadron[1]. Under this assumption, the correction to the total cross section is

$$\alpha C_1 \langle \ln\frac{q_i q_j}{m_i m_j}\rangle \langle n_{ch}\rangle \sigma_{tot}.$$

Making furthermore the assumption of equipartition in rapidity space, we readily obtain $\langle \ln(q_i q_j/m_i m_j)\rangle = (1/3)\ln(s/m_p^2)$. As a final result, the measured total cross section $\sigma_{tot}^{exp}$ is expressed in terms of the purely hadronic $\sigma_{tot}$ by the following relation:

$$\sigma_{tot}^{exp} = (1 + \alpha\frac{C_1}{3}\ln\frac{s}{m_p^2}\langle n_{ch}\rangle)\sigma_{tot}. \tag{2}$$

If we use for the multiplicity the usual approximation $\langle n_{ch}\rangle = a \ln s + b$, $a = 1.65$, then we obtain from (2)[2]