A more detailed analysis of the polaron-phonon interaction will be published later.

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LIMITATION ON THE MASSES OF SUPERCHARGED HADRONS IN THE WEINBERG MODEL

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Upper bounds on the mass of the supercharged quark, which follow from an analysis of the $K_L \to 2\mu$ decay and the K_L - K_S mass difference, are obtained within the framework of a weak-interaction scheme based on hadron SU(4) symmetry.

The need for introducing supercharged hadrons is connected with the requirement that there be no weak neutral currents with $|\Delta S| = 1$ [1]. A renormalizable scheme of weak and electromagnetic hadron interactions, based on introducing a fourth p' quark, was proposed in [2, 3]. We derive below bounds on the masses of the p' quark and the ordinary p quark; these bounds follow from an analysis of the $K_L \to 2\mu$ decay and the K_L - K_S mass difference. The analysis is based on the assumption $m_p \ll m_{p'} \ll \mu_W$, where m_p, $m_{p'}$, and μ_W are the masses of the p quark, the p' quark, and the W boson.

We start with a calculation of the annihilation amplitude of free quarks λ and n into a pair $\mu^+\mu^-$. This amplitude can serve as an effective Lagrangian, the matrix element of which between the states K^0 and the vacuum gives the amplitude of the sought process $K_L \to \mu^+\mu^-$.

In the calculation we have used the so-called generalized ξ-formalism [4]. It reduces to selection of a gauge in which the vector-field propagator is equal to

$$-i\frac{1}{q^2-\mu_W^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu(1-1/\xi)}{q^2-\mu^2/\xi}\right]. \qquad (1)$$

The unphysical pole at $q^2 = \mu^2/\xi$ is cancelled by the contribution of the fictitious scalar particles with mass μ^2/ξ. The final expression for the amplitude, of course, should not depend on ξ.

We note the following curious circumstance: In the usual ξ-formalism [5], which uses the propagator (1) with subsequent transition to the limit $\xi \to 0$, there are no fictitious scalar particles. Yet the contribution made to the discussed amplitude from certain diagrams containing these particles remains finite also as $\xi \to 0$. In the usual ξ-formalism, the absence of such diagrams is compensated for by modification of the vector vertices.

One more remark concerning the calculation technique. Calculation of the $Z\lambda n$ vertex gives rise to divergent diagrams with scalar particles. Although the divergences cancel out mutually, the question is raised of the correctness of the calculation of the finite part of the amplitude. A gauge-invariant method of obtaining the answer may be the Pauli-Villars regularization, wherein one introduces an additional isodoublet of scalar particles with indefinite metric and anticommuting commutation relations. The mass of these particles plays the role of the cutoff parameter and should be allowed to go to infinity.

Another way is to use the Ward identity for the $Z\lambda n$ vertex. A nonzero vertex at a Z-boson momentum $k_\mu = 0$ is then possible only if the current j_μ^Z, the source of the field Z_μ, is not conserved.

By this method, the calculations can be performed also without using the ξ-formalism, directly in the Proca gauge. Taking into account the relation

$$\partial_\mu i_\mu^Z = i m_{p'} \sqrt{g^2 + g'^2}\, \bar{p}' \gamma_5 p' \qquad (2)$$

we find that the contribution of all the diagrams with Z-boson poles is

$$M_1 = \frac{i g^2 m_{p'}}{2\mu_W^2} \bar{\mu} \gamma_\lambda \gamma_5 \mu \int \frac{d^4 q}{(2\pi)^4 (q^2 - \mu_W^2)} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{\mu_W^2} \right) \frac{\partial}{\partial k_\lambda} M_{\mu\nu} \Big|_{k=0}, \qquad (3)$$

where

$$M_{\mu\nu} = - \int d^4x\, d^4y\, e^{-iqx + iky} < T i_\mu^-(x)\, i_\nu^+(0) \bar{p}'(y) \gamma_5 p'(y) >, \qquad (4)$$

θ is the Cabibbo angle, and $j_\mu^\pm$ are the sources of the $W^\pm$-boson fields. All that is connerved is the axial muon current, since it is the only one contributing to the $K_L \to \mu^+\mu^-$ decay.

The contribution made to the amplitude by diagrams with intermediate W^+ and W^- bosons takes the form

$$M_2 = - \frac{i g^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - \mu_W^2)^2} \left(g_{\mu\mu'} - \frac{q_\mu q_{\mu'}}{\mu_W^2} \right) \left(g_{\nu\nu'} - \frac{q_\nu q_{\nu'}}{\mu_W^2} \right) \times$$
$$\times \bar{\mu} \gamma_{\nu'} (1 + \gamma_5) \frac{1}{\hat{q}} (1 + \gamma_5) \mu\, i \int d^4x\, e^{-iqx} < T i_\mu^+(x)\, i_\nu^-(0) >. \qquad (5)$$

Using the commutation relations between j_μ^+, j_μ^- and $\bar{p}'\gamma_5 p'$ we can easily show that the contributions from $q_\mu q_\nu/\mu_W^2$ in (3) and from $q_\mu q_{\mu'} q_\nu q_{\nu'}/\mu_W^4$ in (5) cancel each other, accurate to terms of higher order in $m_{p'}^2/\mu_W^2$ or k^2/μ_W^2.

This cancellation, as well as expressions (3 - 5) themselves, is not connected with the free-quark assumption and holds for arbitrary initial and final hadron states. It must be emphasized, however, that we are dealing with cancellation of diverging quantities, and such a procedure requires in itself an additional definition. The correctness of the performed transformation is guaranteed by the agreement between the final result and the answer obtained in the generalized ξ-formalism.

To find the hadron matrix elements contained in (3) and (4) it is necessary to make additional assumptions. In the free-quark approximation, we arrive at the following amplitude of the transition $\bar{\lambda} n \to \mu^+\mu^-$

$$M = - \frac{G^2 m_{p'}^2 \cos\theta \sin\theta}{4\pi^2} \bar{\lambda} \gamma_\lambda (1 + \gamma_5) n\, \bar{\mu} \gamma_\lambda \gamma_5 \mu. \qquad (6)$$

The result depends neither on the Weinberg mixing angle nor on the electric charges ascribed to the quarks.

We regard (6) as the effective Lagrangian. Then, taking into account the relation

$$< 0 | \bar{\lambda} \gamma_\mu (1 + \gamma_5) n | K^0 > = < 0 | \bar{\lambda} \gamma_\mu (1 + \gamma_5) p | K^+ > = f_K k_\mu, \qquad (7)$$

where k_μ is the K-meson momentum, we obtain

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu)} = \frac{G^2 m_{p'}^4 \cos^2\theta}{2\pi^4}. \qquad (8)$$

Comparing expression (8) with the experimental value of this relation [6], $\sim 4\times10^{-9}$, we obtain the following bound on the mass of the supercharged quark

$$m_{p'} \lesssim 9 \text{ GeV}. \qquad (9)$$

Recognizing that $m_p \neq 0$, we rewrite the bound (9) in the form $\sqrt{m_{p'}^2 - m_p^2} \leq 9$ GeV.

An examination of the Feynman integrals that determine the $\bar{\lambda} n \to \mu^+\mu^-$ amplitude shows that the main contribution is made to them by two regions of integration with respect to the virtual momentum q, namely $m_{p'} \ll q \lesssim \mu_W$ and $q \sim m_{p'}$. Our calculation is correct if the strong interaction can be neglected in these regions. Such an assumption is quite natural for the first region.

We note that the contribution of this region to the $K_L \to 2\mu$ amplitude can be obtained with the aid of the Bjorken asymptotic expansion [7]. It is necessary then, however, to use equal-time current commutators and their time derivatives up to second order, inclusive, which is apparently already very close to the free-quark model.

The assumption that strong interactions are not essential at $q \sim m_{p'}$ is in any case not internally contradictory if $m_{p'} \gg m_p$.

A more stringent bound than (9) on the mass of the supercharged quark can be obtained by estimating the difference of the K_L and K_S meson masses. We confine ourselves again to the free-quark approximation and consider the transition $\bar{\lambda}n \to W^+W^- \to \lambda\bar{n}$. Using the obtained amplitude as the effective Lagrangian, we obtain the estimate

$$m_L - m_S = \frac{2(m_{p'} - m_{p'})^2}{m_\mu^2}\Gamma(K^+ \to \mu^+\nu). \tag{10}$$

Hence

$$m_{p'} - m_p \sim 1 \text{ GeV}. \tag{11}$$

This estimate is less reliable than (9), since it does not take into account the contributions of the intermediate states $W^+ + W^-$ + hadrons. We note that in this case only the region of "small" virtual momenta $q \sim m_{p'}$ is of importance.

Analogous calculations in models of the Giorgi-Glashow type were performed in [8]. The calculations for the $K_L \to 2\mu$ decay are much simpler in these models, owing to the absence of the Z boson.

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ENERGY SPECTRUM OF DISORDERED THREE-DIMENSIONAL SYSTEM

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Bychkov [1] recently examined rigorously the density of the energy spectrum of a one-dimensional system with potential energy in the form $U'(x) = \sum_n \lambda_n \delta(x - x_n)$, where the points x_n are arranged regularly on the X axis, and λ_n are random quantities with a distribution

$$P(\lambda_n) = \frac{\lambda_2}{\pi[(\lambda_n - \lambda_1)^2 + \lambda_2^2]}. \tag{1}$$

The present paper is aimed at generalizing the results of [1] to the case of a three-dimensional lattice with an arbitrarily oriented short-range definite-sign potential V(r). Let the Hamiltonian of the system be

$$H = -\Delta_r + \sum_n \lambda_n V(\mathbf{r} - \mathbf{R}_n). \tag{2}$$