the main contribution is made to them by two regions of integration with respect to the virtual momentum q, namely $m_{p'} \ll q \lesssim \mu_W$ and $q \sim m_{p'}$. Our calculation is correct if the strong interaction can be neglected in these regions. Such an assumption is quite natural for the first region.

We note that the contribution of this region to the $K_L \to 2\mu$ amplitude can be obtained with the aid of the Bjorken asymptotic expansion [7]. It is necessary then, however, to use equal-time current commutators and their time derivatives up to second order, inclusive, which is apparently already very close to the free-quark model.

The assumption that strong interactions are not essential at $q \sim m_{p'}$ is in any case not internally contradictory if $m_{p'} \gg m_p$.

A more stringent bound than (9) on the mass of the supercharged quark can be obtained by estimating the difference of the K_L and K_S meson masses. We confine ourselves again to the free-quark approximation and consider the transition $\bar{\lambda}n \to W^+W^- \to \lambda\bar{n}$. Using the obtained amplitude as the effective Lagrangian, we obtain the estimate

$$m_L - m_S = \frac{2(m_{p'} - m_{p'})^2}{m_\mu^2}\Gamma(K^+ \to \mu^+\nu). \tag{10}$$

Hence

$$m_{p'} - m_p \sim 1 \text{ GeV}. \tag{11}$$

This estimate is less reliable than (9), since it does not take into account the contributions of the intermediate states $W^+ + W^-$ + hadrons. We note that in this case only the region of "small" virtual momenta $q \sim m_{p'}$ is of importance.

Analogous calculations in models of the Giorgi-Glashow type were performed in [8]. The calculations for the $K_L \to 2\mu$ decay are much simpler in these models, owing to the absence of the Z boson.

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ENERGY SPECTRUM OF DISORDERED THREE-DIMENSIONAL SYSTEM

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Submitted 18 April 1973; resubmitted 19 June 1973
ZhETF Pis. Red. 18, No. 2, 145 - 147 (20 July 1973)

Bychkov [1] recently examined rigorously the density of the energy spectrum of a one-dimensional system with potential energy in the form $U'(x) = \sum_n \lambda_n \delta(x - x_n)$, where the points x_n are arranged regularly on the X axis, and λ_n are random quantities with a distribution

$$P(\lambda_n) = \frac{\lambda_2}{\pi[(\lambda_n - \lambda_1)^2 + \lambda_2^2]}. \tag{1}$$

The present paper is aimed at generalizing the results of [1] to the case of a three-dimensional lattice with an arbitrarily oriented short-range definite-sign potential V(r). Let the Hamiltonian of the system be

$$H = -\Delta_r + \sum_n \lambda_n V(\mathbf{r} - \mathbf{R}_n). \tag{2}$$

The summation is over the points λ_n of the three-dimensional lattice, with the distribution (1). The level density $\rho(E)$ is equal to

$$\rho(E) = \frac{1}{\pi V} \operatorname{Im} \operatorname{Sp} < \frac{1}{H - E - i\delta} > , \tag{3}$$

V is the volume of the system.

We express the Green's function $G(r_1, r_2; t)$ of the system with the aid of the Feynman continual integral [2]

$$G(\mathbf{r}_1, \mathbf{r}_2; t) = \int_{\mathbf{r}_1}^{\mathbf{r}_2} d\mathbf{r}(t) \exp i \{ \int_0^t \dot{\mathbf{r}}^2(\tau)\, d\tau + \sum_n \lambda_n \int_0^t V(\mathbf{r}(\tau) - \mathbf{R}_n)\, d\tau \}, \tag{4}$$

and average in (4) explicitly over the distribution (1). The integration yields again an exponential continual integral, the calculation of which is equivalent to a solution of the Schrödinger equation with a complex potential[1)]

$$\tilde{H} = -\Delta_r + \sum_n [\lambda_1 V(\mathbf{r} - \mathbf{R}_n) + i\lambda_2 | V(\mathbf{r} - \mathbf{R}_n)|]. \tag{5}$$

Thus, to determine the system Green's function averaged over (1) it is necessary to solve the usual Schrödinger equation for a regular lattice with an optical one-center potential[2)] $\lambda_1 V(\vec{r}) + i\lambda_2 |V(\vec{r})|$.

By way of example we consider a small-radius potential, for which only s-wave scattering is effective. The problem of scattering by a three-dimensinal lattice under such conditions was considered by Kagan and Afanas'ev [3]. Performing analogous calculations, we obtain

$$\rho(E) = \frac{1}{8\pi^4} \int \frac{d^3 q\, M_2(\mathbf{q}, E)}{[\mathbf{q}^2 - E + M_1(\mathbf{q}, E)]^2 + M_2^2(\mathbf{q}, E)}, \tag{6}$$

where

$$M_1 + iM_2 = \frac{4\pi}{V_0} \frac{1}{-\kappa_1 + i\kappa_2 + D(\mathbf{q}, E)}, \tag{7}$$

$$D(\mathbf{q}, E) = \frac{4\pi}{V_0} \left\{ \sum_{\mathbf{K} \neq 0} \frac{1}{(\mathbf{q} + \mathbf{K})^2 - E} - \frac{V_0}{8\pi^3} \int \frac{d^3 q'}{\mathbf{q}'^2} \right\} \tag{8}$$

Here V_0 is the volume of the cell. The summation in (8) is over the reciprocal-lattice vectors. $\kappa_1 - i\kappa_2$ is connected with $\lambda_1 + i\lambda_2$ through the Schrödinger equation with potential $\lambda_1 V(\vec{r}) + i\lambda_2 |V(\vec{r})|$ in the internal region [4]. If $\chi(r)$ (where $\chi(0)$ is the solution of the equation

$$-\frac{d^2 \chi}{dr^2} + [\lambda_1 V(r) + i\lambda_2 | V(r)|]\chi = 0, \tag{9}$$

then

$$\kappa_1 - i\kappa_2 = \frac{\partial}{\partial r} \ln \chi(r) |_{r = R_0}. \tag{10}$$

R_0 is the effective radius of the potential $V(\vec{r})$. In particular, $\kappa_2 = 0$ when $\lambda_2 = 0$.

Thus, the density of states is expressed in this model in quadratures. The integral (6) can be determined, for example, numerically for a given concrete lattice. From (6) we can draw several general conclusions.

1. If $E > 0$, the density $\rho(E)$ has singularities of the root type at the points $E = \vec{K}^2/4$, where $\vec{K}$ is an arbitrary vector of the three-dimensional reciprocal lattice ($\vec{K} \neq 0$)

$$\rho(E) \approx A_{\mathbf{K}} \sqrt{| E - \mathbf{K}^2/4 |}, \quad E \to \mathbf{K}^2/4, \tag{11}$$

i.e., the level density retains the "memory" of the lattice, just as in the one-dimensional case [1].

2. As $E \to -\infty$ we have

$$\rho(E) \approx \frac{\kappa_2}{2\pi V_0 |E|^{3/2}} . \tag{12}$$

Such an asymptotic form corresponds to a distribution over the binding energy for one isolated well.

3. If $\kappa_1 < 0$ and $|\kappa_1| \to \infty$, then in an ideal lattice (i.e., at $\lambda_2 = 0$), there is a deep band with dispersion

$$\epsilon_a(\mathbf{q}) = -\kappa_1^2 + \frac{4|\kappa_1|}{\pi} \sum_n e^{i\vec{q}\mathbf{R}_n} \frac{e^{-|\kappa_1 \mathbf{R}_n|}}{|\mathbf{R}_n|} . \tag{13}$$

It suffices to sum in (13) over the nearest neighbors. In the nonideal case, the band is smeared out and the density corresponding to it takes the form

$$\rho(E) = \frac{1}{8\pi^4} \int_\Omega \frac{d^3q \, 2|\kappa_1|\kappa_2}{[E - \epsilon_a(\mathbf{q})]^2 + 4|\kappa_1|^2\kappa_2^2} . \tag{14}$$

The integration with respect to $\vec{q}$ is carried out here within one Brillouin zone Ω. It is evident from this expression that there are no singularities of the Van-Hove type [5] in the density of states $\rho(E)$.

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1) It is precisely here that it is required that V(r) be of definite sign.

2) This conclusion is, of course, formal. To obtain this prescription more rigorously it would be necessary to prove that the Green's function $G(r, r', t|\lambda_1, \lambda_2, \ldots, \lambda_4)$ for the Hamiltonian (2), as a function of all the variables λ_i, is analytic in each of the variables in the upper or lower half-plane of the complex λ_i (with all the remaining variables fixed). Such a proof is hardly possible for a potential of arbitrary form. However, the case of one-dimensional δ-like potentials, analyzed in [1], shows that such potentials exist.

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