$$M = xM_A + yM_B = (\nu/2WN)\{xM_A[(U/2z) + xI^{AA} + yI^{AB}] + yM_B[(U/2z) + xI^{AB} + yI^{BB}]\}, \quad (12)$$

where ν is the number of states in the band and N is the number of atoms of the alloy.

Since $\delta_\sigma \ll 1$ in our case, which is equivalent to W >> U, I, the equalities in (12) are compatible only if $M = M_A = M_B = 0$. The limiting case of narrow bands, while a crude approximation, still enables us to observe a possible cause of the observed singularities of invar alloys.

[1] E. I. Kondorsky and V. L. Sedov, J. Appl. Phys. Suppl. 31, 331S (1960).
[2] S. K. Sidorov, A. V. Doroshenko, Fiz. Met. Metallov. 19, 786 (1965); V. E. Arkhipov, A. Z. Men'shikov, and S. K. Sidorov, Zh. Eksp. Teor. Fiz. 61, 1501 (1971) [Sov. Phys.-JETP 34, 799 (1972)].
[3] T. Odagaki and T. Yamamoto, J. Phys. Soc. Japan 32, 104 (1972).
[4] T. Mizocouchi, ibid. 25, 904 (1968).
[5] M. Shimizu and S. Hirooka, Phys. Lett. 27A, 530 (1968); 30A, 133 (1969).
[6] M. Collins and G. G. Low, Proc. Phys. Soc. London 86, 535, 973 (1965).
[7] I. M. Puzei, V. I. Goman'kov, and L. A. Loshmanov, Fiz. Met. Metallov. 25, 36 (1968). Izv. AN SSSR ser. fiz. 28, 440 (1964).
[8] P. Soven, Phys. Rev. 156, 809 (1967).
[9] B. Velicky, S. Kirpatrick, and H. Ehrenreich, Phys. Rev. 175, 747 (1968).
[10] F. Brouers, A. V. Vedyayev, and M. Giorgino, Phys. Rev. B7, 380 (1973).

ANOMALIES OF THE ELECTRONIC MAGNETIC SUSCEPTIBILITY OF A PLATE

M. I. Kaganov and S. S. Nedorezov
Institute of Physics Problems, USSR Academy of Sciences; Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted 10 July 1973
ZhETF Pis. Red. 18, No. 5, 330 - 332 (5 September 1973)

It is predicted that the diamagnetic susceptibility of a plate is subject to quasiperiodic temperature-independent oscillations due to electrons whose orbits are tangent to both plate boundaries. The calculation is performed in the one-electron approximation of band theory.

For a plate of thickness L, we investigated theoretically the dependence of the diamagnetic susceptibility χ on the magnetic field H parallel to the plate boundaries. The calculation was within the framework of the one-electron approximation of band theory in the quasiclassical approximation. We predict the existence of unique quasiperiodic oscillations of χ, which depend little on the temperature. The oscillations are due to internal (not Fermi) electrons. The oscillations should be observed not only in metals, but also in dielectrics with sufficiently broad bands.

Quantization of the electron energy in a magnetic field leads to a complicated dependence of the thermodynamic potential Ω of the electron gas on the magnetic field. Confining ourselves to the quasiclassical approximation (see below), we can say that each quantized orbit of the electron adds to Ω a contribution that contains a periodic function with argument $cS/e\hbar H$, where $S = S(\varepsilon, p_z, p_x)$ is the area inside the orbit traced by an electron having an energy ε and momentum projections p_x and p_z in momentum space (or part of the trajectory lying in the plate). The axes are chosen such that $H_z = H$ and $H_x = H_y = 0$; the y axis is perpendicular to the plate boundaries; the plate occupies the strip $0 < y < L$. Summation (integration) over all orbits results in contributions to the oscillating part of Ω (which we denote δΩ) from orbits of electrons having an energy equal to the Fermi energy and an extremal area S, i.e., the electrons for which $\partial S/\partial p_z = 0$ (the de Haas - van Alphen effect, which is observed in metals and in degenerate semiconductors [1]). In the case of plates, however, δΩ receives contributions not only from these orbits, but also from orbits that are tangent to the two plate boundaries, and naturally satisfy the quantization condition

$$S(\varepsilon, p_z) = (2\pi\hbar eH/c)(n + 5/6), \quad n = 1, 2, 3, \ldots \quad (1)$$

The quantity p_x, which determines the position of the orbit in coordinate space, was left out of formula (1) and was chosen to satisfy the condition of tangency to both boundaries of the plate; the constant 5/6 was obtained from an analysis of the quasiclassical motion of the electron along nearly tangential orbits. This question, as well as a derivation of the equation for (H), will be dealt with in a detailed article[1]. We note only that it was assumed in the calculations that the electron is specularly reflected from the plate boundary. Since we are interest in tangential orbits, this assumption is justified (see [3]).

In the case of a quadratic dispersion it follows from the compatibility of the quantization condition (1) and the tangency condition

$$D_{\tan}(\epsilon, p_z) = LeH/c \qquad (2)$$

(D is the dimension of the orbit along p_x) that the argument of the periodic part is equal to $\pi L^2 eH/4\hbar c$ and does not depend on the energy at all. This means that $\delta\Omega$ contains a periodic function with a period

$$\Delta H = 8\hbar c/eL^2. \qquad (3)$$

Since the period does not depend on the energy, it is clear that all the electrons whose maximum orbit diameter (relative to p_z) is larger than the thickness of the film and whose centers are located at $y = L/2$ take part in these oscillations. The physical meaning of the periodic dependence is particularly simple in this case: when H is changed by ΔH, the number of magnetic flux quanta within the tangent orbit changes by unity.

The quasiclassical approximation is valid if

$$8\hbar c/eL^2 << H \lesssim 2\pi\hbar c/aeL; \qquad L >> a; \qquad (4)$$

where a is the interatomic distance. The upper bound of H corresponds to the condition that the dimension of the closed orbit in momentum space must not exceed the dimension $2\pi\hbar/a$ of the reciprocal-lattice cell.

An estimate of the oscillation amplitude shows that $\delta\chi \simeq \chi_L (H/\Delta H)^{4/3}$, where χ_L is the Landau diamagnetic electronic susceptibility [4]. According to (4) we have $\delta\chi >> \chi_L$, which seems to make the effect observable.

The particular simplicity of the effect in the case of quadratic dispersion is due to the fact that S and D depend on the same combination of ϵ and p_z. For an arbitrary dispersion law this is of course not the case. The entire picture of the oscillations becomes more complicated. The condition (1) written for the extremal (with respect to p_z) area S determines, together with the tangency condition (2), the energies of the electrons that take part in the oscillations considered here. Instead of a rigorous periodic dependence, as in the case of quadratic dispersion, $\delta\chi$ has a complicated oscillatory dependence on the magnetic field. The period is only of the same order of magnitude of ΔH ($\delta\chi = \sum A_i(H) \ln|2\sin[cS_{max}(\epsilon_i)/2\hbar eH) - (5\pi/6)]|$, and the value of the amplitude $A_i(H)$ is not written out here). As before, the oscillations are not governed by the Fermi electrons[2].

In conclusion, we note the following: 1)Since the predicted oscillations are determined by the inner electrons, their amplitude is practically independent of the temperature. 2) The effect described here should be observed not only in metallic plates, but also in dielectric ones, provided that the one-electron approximation is valid for the filled bands. 3) Investigations of plates make it apparently possible to determine the ground-state structure of the band electrons of crystals.

The authors take the opportunity to thank I. M. Lifshitz for interest in the work and for stimulating discussions, and L. A. Fal'kovskii for critical remarks.

[1] A. M. Kosevich and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 743 (1955) [Sov. Phys.-JETP 2, 646 (1956)].
[2] S. S. Nedorezov, ibid. 64, 624 (1973) [37, No. 2 (1973)].
[3] A. F. Andreev, Usp. Fiz. Nauk 105, 113 (1971) [Sov. Phys.-Usp. 14, 609 (1972)].
[4] L. D. Landau, Z. Physik 64, 629 (1930).

1) For an analysis of a smooth $\Omega(H)$ dependence see [2].

2) The role of open trajectories calls for a separate analysis.