ATOMS IN A SUPERSTRONG MAGNETIC FIELD

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Tremendous magnetic fields on the order of 10^{12} G, which exist according to present concepts in neutron stars, can greatly change the physical properties of matter, particularly those of isolated atoms [1, 2]. One of the authors [1] has shown that in the magnetic field interval $Z^{4/3} < B < Z^3$ (Z - atomic number, B - magnetic field in atomic units of $m^2e^2ch^{-3} = 2.35 \times 10^9$ G), the ground state of a very heavy atom can be described within the framework of the modified Thomas-Fermi model. The atom retains in this case spherical symmetry, and its radius varies like $Z^{1/5}B^{-2/5}$. The same article contains qualitative ideas concerning the possible conservation of spherical symmetry also when $B > Z^3$, when not more than one electron remains at each level. However, the latter statement turns out to be incorrect - the more accurate quantitative analysis presented below for the limiting case $B > Z^3$ shows that in this case the ground state with minimum energy corresponds not to a spherical electron density distribution, as in the case when $B < Z^3$, but to a distribution strongly elongated in the direction of the magnetic field. In this sense the case $B \gg Z^3$ corresponds to the hydrogen atom at $B \gg 1$ [3 - 6].

When $B \gg Z^3$, the levels that are lowest with respect to the azimuthal quantum number m are occupied in the ground state by not more than one electron per level [1]. In the self-consistent-field approximation, the energy of an atom or an ion in a superstrong field is determined by the expression (in atomic units):

$$E = \int \left[\frac{1}{2} \sum_i \left(\frac{\partial \Psi}{\partial z_i} \right)^2 - \sum_i \frac{Z}{r_i} \Psi^2 + \frac{1}{2} \sum_{i \neq j} \frac{1}{r_{ij}} \Psi^2 \right] \prod_i dr_i, \tag{1}$$

where $\vec{r}_i$ is the coordinate of the j-th electron, r_j is its distance from the nucleus, $r_{ij} = |\vec{r}_i - \vec{r}_j|$, and the wave function Ψ is chosen in the form of an antisymmetrized product of single-particle wave functions in the form $e^{im\theta}R_m(\rho)\Psi_{Om}(z)$, with the radial part R_m corresponding to the lower Landau level (ρ, θ, and z are the cylindrical coordinates), and Ψ_{Om} corresponds to the lower level of the longitudinal motion for the given m. In expression (1), as we see, there remain only the kinetic energy of the longitudinal motion and the potential energy. We neglect the exchange correction and replace Ψ simply by the product of the single-particle wave functions in the self-consistent electric field. Expression (1) then goes over into a sum of integrals with respect to the single-particle wave functions, and E can be expressed in terms of the average electron density $n = \sum_i \Psi_j^2$. We define the average wave function $\tilde{\psi}$ by means of the relation $\tilde{\psi}^2 = n/N$; from (1) we obtain

$$E = N \int \left[\frac{1}{2} \left(\frac{\partial \tilde{\psi}}{\partial z} \right)^2 - \frac{Z}{r} \tilde{\psi}^2 + \frac{N-1}{2} \left\langle \frac{1}{r_{12}} \right\rangle \tilde{\psi}^2 \right] dr, \tag{2}$$

where

$$\left\langle \frac{1}{r_{12}} \right\rangle = \int \frac{1}{|r - r_2|} \tilde{\psi}^2(r_2) dr_2.$$

The energy of the ground state is determined from the condition of the minimum of the functional (2) under the additional condition $\int\tilde{\psi}^2 dr = 1$, and the occupation numbers do not exceed unity. The radial wave function $R_m(\rho)$ is localized relatively close to $\rho_m = \sqrt{2m/B}$, and when all the shells from m = 0 to m = N are filled, the electron density is bounded by a cylinder of radius $\rho_N \simeq \sqrt{2N/B}$, i.e., $n(\rho, z)$ decreases rapidly when $\rho > \rho_N$. We choose a trial function in the form[1])

$$n = \tilde{\psi}^2 = \frac{\alpha}{\pi\rho_N^2}\exp(-2\alpha|z| - \rho^2/\rho_N^2), \tag{3}$$

where α is a parameter determined from the condition that E be a minimum.

At very large B, we have $\alpha\rho_N \ll 1$, i.e., the electron cloud is strongly elongated in the direction of the magnetic field. Taking this into account, we obtain with logarithmic accuracy, after substituting (3) in (2),

$$E = \frac{N\alpha^2}{2} - 2NZL\alpha + \frac{1}{2}N(N-1)L\alpha, \text{ where } L = \ln\frac{1}{\alpha\rho_N} \gg 1. \tag{4}$$

Expression (4) can be obtained in a simple manner from the following considerations. The second term in (4) corresponds to averaging of the quantity $1/r \simeq 1/z$ with the running density $\lambda = \alpha\exp(-2\alpha|z|)$ and with cutoff of the logarithmic divergence for small z at a value $z_{min} \sim \rho_N$. In the third terms of (2) and (4), the quantity $\langle 1/r\rangle$ can be regarded as the potential ϕ at the point z of a thin charged linear body of radius ρ_N with a running density λ. But the potential ϕ is determined, with logarithmic accuracy, by the local value of the running charge density λ, and is equal to $\phi = 2\lambda\ln(1/\alpha\rho_N)$, where $1/\alpha$ is the characteristic length of the charge distribution. Averaging this potential with weight λ gives the third term in (4). From the condition of the minimum of (4) with respect to α, assuming approximately that L is constant, we obtain, with logarithmic accuracy,

$$\alpha = \frac{1}{2}(4Z - N + 1)L; \qquad E = -\frac{N}{8}L^2(4Z - N + 1)^2. \tag{5}$$

From the known α we obtain, with logarithmic accuracy, the value of L at N = Z

$$L \cong \frac{1}{2}\ln\frac{B}{Z^3}. \tag{6}$$

An estimate of the binding energy in accordance with the spherical model [1] would yield $E \sim Z^3$, which is smaller by a factor L^2 than (5). Thus, when $B \gg Z^3$, a lower energy is possessed by an electron cloud elongated in the direction of the magnetic field. We note that when Z = N = 1 expression (1) coincides, with logarithmic accuracy, with the energy of the ground state of the hydrogen atom [6]: $E = -(1/2)(\ln B)^2$, so that formula (5) is suitable for small z.

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[1])The exponential dependence of ψ on z corresponds to a "deep" ground level.

From (5) we obtain the ionization energy of a neutral atom $E_i = -\partial E/\partial N|_{N=Z}$:

$$E_i = \frac{3}{8}L^2Z^2. \qquad (7)$$

As seen from (7), in the limiting case $B >> Z^3$ the ionization energy increases very rapidly with Z, unlike the results of [2], where the ionization energy at $B = 2 \times 10^{12}$ G is approximately constant in the interval $1 < Z < 10$. This is due to the fact that the parameters in [2] pertain to the intermediate region $B \sim Z^3$, and the asymptotic formula (7) is possibly not yet applicable. We note that a correction for the exchange interaction turns out to be small compared with (7) at large Z [2].

It follows from (5) that at $N > Z$ the energy E decreases with N. Therefore the formation of negative ions is energetically favored up to $N \approx 4Z/3$, where $\partial E/\partial N = 0$. Even more favored, at not too high temperatures, is the formation of molecules with large binding energy. In addition, inasmuch as the atoms that are strongly elongated at $B >> Z^3$ have a large quadrupole moment, their interaction energy should be very large, and apparently even at temperatures $\sim 10^6$ deg heavy matter in a superstrong field can become condensed into a solid phase even on the surface of a pulsar. This will be considered separately.

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DIFFUSION MOBILITY OF NEGATIVE IONS IN SOLID HELIUM

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The purpose of this article is to calculate the mobility of negative ions under the assumption that this mobility has a diffusion origin. An electron in a vacuum bubble of radius $a \approx 10$ Å [1] begins to exert an asymmetrical pressure on the bubble walls when an external electric field of intensity E is turned on. This pressure, after a certain adjustment process, leads to a stationary diffusion flow of vacancies from high-pressure to low-pressure areas. This flow causes the bubble to move as a unit in the direction of the driving field $\vec{E}$. Diffusion problems of this type were encountered already, for example, in [2] in the study of the diffusion-viscous flow of polycrystals under the influence of pressures applied from the outside. Therefore the system of equations which we need and its proofs can all be taken from these papers.

The stationary volume field of the vacancies $c(\vec{r})$ is described by the harmonic equation[1)]

[1)]We neglect surface diffusion over the surface of the ion, since the surface layer of the ion is under a large spherically-symmetrical electron pressure, and consequently the surface diffusion cannot greatly exceed the volume diffusion.