From (5) we obtain the ionization energy of a neutral atom $E_i = -\partial E/\partial N|_{N=Z}$:

$$E_i = \frac{3}{8}L^2Z^2. \qquad (7)$$

As seen from (7), in the limiting case $B >> Z^3$ the ionization energy increases very rapidly with Z, unlike the results of [2], where the ionization energy at $B = 2 \times 10^{12}$ G is approximately constant in the interval $1 < Z < 10$. This is due to the fact that the parameters in [2] pertain to the intermediate region $B \sim Z^3$, and the asymptotic formula (7) is possibly not yet applicable. We note that a correction for the exchange interaction turns out to be small compared with (7) at large Z [2].

It follows from (5) that at $N > Z$ the energy E decreases with N. Therefore the formation of negative ions is energetically favored up to $N \approx 4Z/3$, where $\partial E/\partial N = 0$. Even more favored, at not too high temperatures, is the formation of molecules with large binding energy. In addition, inasmuch as the atoms that are strongly elongated at $B >> Z^3$ have a large quadrupole moment, their interaction energy should be very large, and apparently even at temperatures $\sim 10^6$ deg heavy matter in a superstrong field can become condensed into a solid phase even on the surface of a pulsar. This will be considered separately.

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DIFFUSION MOBILITY OF NEGATIVE IONS IN SOLID HELIUM

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The purpose of this article is to calculate the mobility of negative ions under the assumption that this mobility has a diffusion origin. An electron in a vacuum bubble of radius $a \approx 10$ Å [1] begins to exert an asymmetrical pressure on the bubble walls when an external electric field of intensity E is turned on. This pressure, after a certain adjustment process, leads to a stationary diffusion flow of vacancies from high-pressure to low-pressure areas. This flow causes the bubble to move as a unit in the direction of the driving field $\vec{E}$. Diffusion problems of this type were encountered already, for example, in [2] in the study of the diffusion-viscous flow of polycrystals under the influence of pressures applied from the outside. Therefore the system of equations which we need and its proofs can all be taken from these papers.

The stationary volume field of the vacancies $c(\vec{r})$ is described by the harmonic equation[1)]

[1)]We neglect surface diffusion over the surface of the ion, since the surface layer of the ion is under a large spherically-symmetrical electron pressure, and consequently the surface diffusion cannot greatly exceed the volume diffusion.

$$\Delta c = 0 \tag{1}$$

with the boundary condition

$$c_{r=a} = \frac{c_0 \omega_0}{kT} p_n .$$

p_n are the normal pressures on the surface of the ion of radius a, c_0 is the equilibrium vacancy concentration, ω_0 is the volume of one vacancy, k is Boltzmann's constant, and T is the temperature. Normal vacancy flow to the surface of the ion determines the local rate of displacement of an element of the boundary surface of the negative ion $u_n(\theta)$

$$D \frac{\partial c}{\partial r}\Big|_{r=a} = v_n(\theta) \tag{2}$$

D is the vacancy diffusion coefficient.

In order for the ion to be able to move as a unit without being deformed, it is necessary to satisfy the condition $u_n(\theta) = V_0 \cos\theta$, where V_0 is the velocity of the ion as a whole. Thus, by determining the pressures on the surface of the ion and solving the harmonic problem (1) - (2), we can connect V_0 with the intensity of the applied electric field, i.e., we can find the mobility of the ion $\mu = V_0/E$.

The electron pressure on the surface of the ion p^I_{el} can be obtained with the aid of the following formulas

$$(p'_{эл})_l = P_{lk} n_k ,$$

$$P_{lk} = \frac{\hbar^2}{4m}\left[\frac{\partial\psi}{\partial x_l}\frac{\partial\psi^*}{\partial x_k} - \psi\frac{\partial^2\psi^*}{\partial x_l\,\partial x_k} + \text{к.с.}\right] . \tag{3}$$

Here $\psi(r, \theta)$ is the solution of the Schroedinger equation for the electron in a spherical potential well, given by the boundary condition $\psi(r, \theta)_{r=a} = 0$, in the presence of a constant perturbing electric field of amplitude E. As a result, in first order of perturbation theory, we have for p^I_{el} the expression

$$p'_{эл} = \frac{\pi\hbar^2}{ma^2(2\pi a)^{1/2}} \sum_{l} \frac{E_{0l}}{\lambda_0 - \lambda_l} \frac{\partial}{\partial r} j_l\left(\frac{\pi\beta_l r}{a}\right) P_l(\cos\theta)\Big|_{r=a} \tag{3a}$$

$$l = 1, 3, 5, \ldots$$

m is the electron mass, E_{0l} are the matrix elements of the potential of the perturbing electric field in terms of the wave eigenfunctions of the given problem, λ_l are the energy levels of the unperturbed problem, $P_l(\cos\theta)$ are Legendre polynomials, and $j_l(\pi\beta_l r/a)$ are spherical Bessel functions. The coefficients β_l are determined by the condition $j_l(\pi\beta_l) = 0$.

Expression (3a) for p^I_{el} contains all the odd angle harmonics $P_l(\cos\theta)$.

Only the first of these harmonics, proportional to cos θ, maintains constant motion on the ion along the field. All the remaining harmonics violate the spherical shape of the ion, and therefore should be cancelled out in time by the additional Laplace and electron pressures p_σ and p_{el}^{II} occurring on the surface of the ion when the latter is deformed. Thus, assuming the excess pressure on the surface of the ion to be $p_{el}^{I} + p_\sigma + p_{el}^{II}$, relating the first harmonic of the diffusion problem (1) - (2) with the first harmonic p_{el}^{I}, and stipulating that all the diffusion fluxes with higher harmonics vanish (this condition determines the pressures p_σ and p_{el}^{II}), we arrive at the following final formulas for V_0 and the small deformation of the ion $\xi(\theta)$ ($a(\theta) = a[1 + \xi(\theta)]$). The connection between $\xi(\theta)$ on the one hand, and p_σ and p_{el}^{II} on the other, has the same form as in the calculation of the natural frequencies of a negative bubble in liquid helium [3]

$$V_0 = D\frac{c_0\omega_0}{kT}\frac{\pi\hbar^2}{ma^5}\frac{\Lambda}{|\lambda_0 - \lambda_1|}eE,$$

$$\Lambda = \frac{2\beta_1\int_0 j_0(\pi x)\, j_1(\pi\beta_1 x)\, x^3\, dx}{j_0(\pi\beta_1)\, j_2(\pi\beta_1)}\frac{\partial}{\partial x} j_1(\pi\beta_1 x)\Big|_{x=1}, \quad \beta_1 = 1,4303,$$

$$\xi(\theta) = \sum_l \xi_l P_l(\cos\theta), \qquad \xi_l = \frac{(p_{el}^{I})_l}{\frac{\sigma}{a}[4\pi s_l + l(l+1) - 2]},$$

$$s_l = \frac{j_l'(\pi)}{j_l(\pi)} - \frac{j_0''(\pi)}{j_0'(\pi)}, \qquad l = 3, 5, 7 \ldots$$

Recognizing that in order of magntidue we have $\Lambda \sim 1$, $|\lambda_0 - \lambda_1| \simeq \pi^2\hbar^2/2ma^2$, $a \simeq 10^{-7}$ cm, $\omega_0 \simeq 10^{-22} - 10^{-23}$ cm^3, and the coefficient of self-diffusion is $c_0 D \sim 10^{-7} - 10^{-8}$ cm^2/sec [4], we find that at T = 2°K we have $\mu \simeq 5 \times 10^{-6}$ cm^2/V-sec. This is in qualitative agreement with the recently obtained experimental data on the mobility of negative ions in solid helium (see [5]).

Let us point out also the limitation to which the external electric field is subject if the results are to be applicable. This limitation reduces to the inequality $V_0 \ll D/a$, or, taking into account the expression obtained for V_0,

$$c_0\frac{\omega_0}{a^3}\frac{eEa}{kT} \ll 1.$$

For the fields $E \sim 10^4$ V/cm used in the experiments of [5], this inequality is satisfied with good margin.

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