magnetic surfaces by the presence of the current I_{PS}.

Further increase of the plasma pressure leads rapidly to violation of the Kruskal-Shafranov criterion

$$q(r) = \frac{B_0}{B_I(r)}\frac{r}{R} > 1. \tag{5}$$

The numerical coefficients depend on the details of the distribution of the plasma pressure over the pinch cross section. The limiting value of $\beta = 4\pi p/B_0^2$. obtained by substitution in this criterion, was calculated by us for two different pressure and current distributions at uniform temperature:

$$1)\ p = p_0\left(1 - \frac{r^2}{a^2}\right),\quad B_I^2 = 0{,}81\left(\frac{r}{a}\right)^{5/2} 4\pi p_0 \epsilon^{1/2},\quad q^2(a) = 1{,}23\epsilon^{3/2}/\beta > 1,$$

$$2)\ B_I = B_I(a)\frac{r}{a},\quad 4\pi p = 1{,}48 B_I^2(a)\left[1 - \left(\frac{r}{a}\right)^{3/2}\right],\quad q^2(a) = 1{,}48\epsilon^{3/2}/\beta > 1.$$

In the case of a non-isothermal plasma ($T_e \gg T_i$) with identical temperature and density profiles ($n(r) \sim T_e(r)$), the limiting pressure increases by a factor of three.

We note that the limiting values of β_I turn out to be lower than in the MHD equilibrium model (see [1]).

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CONCERNING ONE MORE METHOD OF MAGNETIC ENERGY PUMPING INTO A TURBULENT PLASMA

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1. If the intensity of plasma waves (plasmons) is sufficiently large, then interactions of the plasmons with one another can become more probable than interactions of plasmons with plasma particles. Under these conditions the plasma can be regarded as consisting of two weakly-interacting subsystems - particles and plasmons - between which a slow energy exchange takes place. The relaxation of the plasma will then have a two-step character - first quasi-static equilibriums will be established in the particle and plasmon subsystems

with different temperatures, followed by a slower process of temperature equalization.

In this communication we wish to point out that if the initial plasmon energy is sufficiently high then it is possible to increase the plasmon energy by the simple method of modulating the external parameters on which their frequencies depend. In the case of a collisionless magnetoactive plasma with hot electrons and cold ions, three types of low-frequency weakly-damped collective oscillations can exist - Alfven, fast, and slow magnetosonic waves. The frequencies of these waves depend on the external magnetic field, and by modulating it we can heat the gas of the Alfven and magnetosonic waves.

The energy imparted to the plasmons in such a heating method (it can be called the method of magnetic pumping) can greatly exceed the Joule heat directly obtained by the particles in the magnetic pumping. The energy acquired by the plasmons goes over gradually to the particles, and a certain stationary energy level (the turbulent-noise level) is established in the plasmon subsystem itself. In a non-isothermal plasma, it is mainly the plasma electrons that become heated in this case (owing to the Landau-damping effect).

In order for the plasmon concept to have a meaning, the plasmon frequencies must be much larger than their reciprocal lifetimes. If this condition is satisfied, it is possible to introduce the number j of plasmons N of different sorts with corresponding frequencies ω_j and wave vectors $\vec{k}_j$, and to investigate their variations resulting from modulation of the frequency and of the collisions between the plasmons. If the modulation is sufficiently slow (the modulation period is much larger than the plasmon relaxation time), then the plasmon distribution functions are stationary, i.e., the changes in the number of plasmons due to the collisions $(N_j)_c$ and due to the frequency modulation $(N_j)_b$ cancel each other

$$(\dot{N}_j)_c + (\dot{N}_j)_b = 0. \tag{1}$$

As a result, the plasma distribution differs from the equilibrium value; on the other hand, a distribution density constantly maintained by the modulation of the magnetic field leads to a growth of the entropy of the plasmon gas, i.e., to its heating[1].

We confine ourselves to the processes in which only three plasmons take part [2]. Since the plasmons obey Bose-Einstein statistics, the change in the number of plasmons per unit time, due to their decay and their coalescence, can be written in the form

$$(\dot{N}_1)_c = \sum_{2,3} \{ w_{j_1 \rightleftarrows j_2 + j_3} [(N_1 + 1) N_2 N_3 - N_1 (N_2 + 1)(N_3 + 1)] + 2 w_{j_1 + j_3 \rightleftarrows j_2} [(N_1 + 1) N_2 (N_3 + 1) - N_1 (N_2 + 1) N_3] \}, \tag{2}$$

where $w_{j_1 \rightleftarrows j_2 + j_3}$ is the probability of the process of decay (coalescence) of the type $j_1 \rightleftarrows j_2 + j_3$, in which the energy-momentum conservation laws $\omega_1 = \omega_2 + \omega_3$ and $\vec{k}_1 = \vec{k}_2 + \vec{k}_3$ are satisfied (ω_α is the frequency of the plasmon of sort j_α with wave number $\vec{k}_\alpha$; the summation is over $\vec{k}_2$ and $\vec{k}_3$ and over the

[1] A similar mechanism, as first shown by one of the authors [1], causes absorption of sound in dielectrics and metals, namely, the absorption of the sound in these substances is due to the modulation of the phonon frequency and of the electron energy by the acoustic field and to a change in the particle distribution due to such a modulation.

plasmon sorts). The collision integral (2) vanishes for a Planck distribution function, which in the case of low-frequency oscillations goes over into the Rayleigh-Jeans formula $N_{\alpha 0} = T^*/\omega_\alpha$ (T^* is the plasmon temperature). Using the well-known expression for the entropy of the boson gas, we can find the time variation of the plasmon entropy

$$\dot{S}^* = \sum_{1,2,3} w_{l_1 \rightleftarrows l_2 + l_3} \ln \frac{(N_1+1)N_2N_3}{N_1(N_2+1)(N_3+1)} \geqslant 0. \tag{3}$$

3. The values of w can be obtained in the magnetohydrodynamic approximation [3 - 4]. The simplest is the case of a small gas-kinetic pressure, when it suffices to take into account only processes in which slow magnetosonic waves take part. If the external magnetic field $\vec{B}$ is subject to slow and shallow modulation, $\vec{B} = \vec{B}_0(1 + b \cos \Omega t)$, $b \ll 1$, then the average energy absorbed by the plasmons per unit time (and per unit volume) is

$$q^* = \overline{T^* \dot{S}^*} \cong \frac{\rho V_A^2}{\omega_i} b^2 \Omega^2 , \tag{4}$$

where ρ is the plasma density, ω_i is the ion cyclotron frequency, and V_A is the Alfven velocity. If the electric conductivity of the plasma is sufficiently large ($\sigma \gg \omega_i (c/V_A)^2$), then the Joule heat acquired by the plasma when the magnetic field is modulated is much smaller than q^*.

Owing to the interaction of the waves with the plasma particles (principally with the electrons), the plasmon energy will tend to a certain stationary value; the energy transferred to the particles in the unit time will then be a*.

4. The proposed magnetic-pumping method can be realized if the following conditions are satisfied

$$\omega_i \gg 1/\tau_j \gg \Omega, \quad \min(1/\tau_j) \gg \max(\gamma_j), \tag{5}$$

where γ_j is the average value of the damping coefficients of the considered waves (as a result of the interaction with the particles), and τ_j is the average lifetime of these waves (as a result of their interaction with one another). By using these inequalities, we can show that the initial energy density of the plasmons W_0 should satisfy the inequality

$$(W_0/nT_e)^2 \gg (m_e/m_i)^{3/2} (\nu/\omega_i), \tag{6}$$

where nT_e is the energy density of the plasma electrons and ν is the frequency of the Coulomb collisions. The energy density of the plasmons in the stationary regime is

$$\bar{W} \sim nT_e (m_i/m_e)^{1/2} (V_A/V_s)^2 (\Omega/\omega_i)^2 b^2. \tag{7}$$

(V_s is the speed of sound).

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SELF-FOCUSING OF A VECTOR FIELD

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Volkov [1], using plasma as an example, has shown in 1958 that nonlinear action of an electromagnetic field on the medium can give rise to spatially-localized distributions of the field, which subsequently were called self-trapping or self-focusing. The nonlinear solution obtained by Volkov for the field equation pertains to the case of a scalar field. In this communication we show that self-focused field distributions exist also for TM waves, when there are two components of the electric field (the vector case). However, such distributions differ qualitatively from the case of a scalar field.

For an electric field in the form

$$\mathbf{E}(r, t) = \mathbf{E}^+(r)\cos\omega t + \mathbf{E}^-(r)\sin\omega t$$

we write down (compare with [2]) the following equations of nonlinear electrodynamics

$$-\Delta\mathbf{E}^{\pm} + \mathrm{grad\,div}\,\mathbf{E}^{\pm} = k^2\epsilon\mathbf{E}^{\pm},$$

where $k^2 = (\omega/c)^2, \quad \epsilon = \epsilon[\omega, (\mathbf{E}^+)^2 + (\mathbf{E}^-)^2].$

For the case, which does not violate the generality, when there is no field energy flux along the x axis, assuming

$$E^+_{x,z} + iE^-_{x,z} = E_{x,z}(x)\exp(ik_z z + i\delta_{x,z}),$$

taking $\delta_z - \delta_x = \pi/2$ and assuming δ_z and δ_x to be constant, we have therefore

$$E_z'' - k_z E_x' + k^2\epsilon[\omega, E_x^2 + E_z^2]E_z = 0, \tag{1}$$

$$k_z E_z' - k_z^2 E_x + k^2\epsilon[\omega, E_x^2 + E_z^2]E_x = 0. \tag{2}$$

Let us consider first the solutions of the system (1) - (2) in the limit of $k_z = 0$. Then, obviously, two different situations are possible. In the first case, the longitudinal component of the electric field is $E_x = 0$, and a solution of the Volkov type is obtained for E_z. In particular, when

$$\epsilon = \epsilon_0 + \Delta(E_x^2 + E_z^2), \quad \Delta > 0, \quad \epsilon_0 < 0 \tag{3}$$

we have [3]

$$E_z = \sqrt{-\frac{2\epsilon_0}{\Delta}}\Big/\mathrm{ch}\,[\sqrt{-\epsilon_0}\,k(x - x_0)], \quad E_x = 0. \tag{4}$$

The other solution of this system (1) - (2) corresponds to the presence of a longitudinal electric field $E_x \neq 0$. This is possible if

$$\epsilon[\omega, E_x^2 + E_z^2] = 0, \tag{5}$$