SPLITTING OF A PHOTON IN A MAGNETIC FIELD AND POLARIZATION OF HARD RADIATION FROM PULSARS

D.V. Gal'tsov and V.V. Skobelev
Submitted 21 December 1970
ZhETF Pis. Red. 13, No. 3, 173 - 175 (5 February 1971)

Starting from CP-invariance considerations and from the energy-momentum conservation law, with allowance for the polarization of vacuum, Adler et al. [1] have shown that the process $\gamma \to 2\gamma$ in a magnetic field close in magnitude to $B_0 = 4.41 \times 10^{13}$ G should lead to polarization of hard γ radiation ($\omega \sim m$, m - electron mass). The effect can be realized under the conditions in neutron stars, if the assumption that superstrong magnetic fields are present in them is valid [2 - 4].

To estimate the probability of photon splitting, they calculated in [1] the contribution made from a sixth-order diagram, including triple interaction with a homogeneous magnetic field. In the opinion of the authors of [1], the contribution of a diagram of order lower than the fourth should be equal to zero. Such a conclusion is based on the premise that to ensure gauge invariance the matrix element must be constructed only of tensors $F_{\mu\nu}^{(i)}$ (i = 1, 2, 3) of the three photons participating in the process, of the tensor $F_{\mu\nu}$ of the constant and homogeneous external field, and of the wave vectors of the photons (in fourth order this construction turns out to be identically equal to zero).

We wish to emphasize that owing to the specific kinematics of the $\gamma \to 2\gamma$ process (the momenta of all three photons are parallel) the gauge invariance in fact imposes much weaker limitations on the possible form of the matrix element, and consequently the conclusions in [1] are inconsistent. The main contribution to the probability of the process in question is made precisely by a fourth-order diagram, and therefore the quantitative estimates made in [1] are too low by many orders of magnitude.

We have calculated the fourth-order diagram and obtained the following expression for the matrix element[1)]

$$M = \frac{7}{6\sqrt{\pi}}\frac{e^4}{m^2}k^\mu F_{\mu\nu}(e^{\nu}(e_1e_2)^* - \lambda e_1^{*\nu}(ee_2^*) - (1-\lambda)e_2^{*\nu}(ee_1^*)). \qquad (1)$$

Here k^μ is the momentum of the primary photon, λk^μ and $(1-\lambda)k^\mu$ $(0 < \lambda < 1)$ are the momenta of the produced photons; e^ν, e_1^ν, and e_2^ν are the corresponding polarization vectors. Taking into account the equality $(ek) = (e_1k) = (e_2k) = 0$ and the antisymmetry of the tensor $F_{\mu\nu}$, we can easily verify that the matrix element (1) is gauge-invariant.

In the case of a homogeneous magnetic field, the probability of the process is

$$dw = w_0\,[(eN)(e_1e_2)^* - \lambda(e_1^*N)(ee_2^*) - (1-\lambda)(e_2^*N)(ee_1^*)]^2\,d\lambda \qquad (2)$$

$$w_0 = (a^3/\pi^2)(49/1152)(B\sin\theta/B_0)^2(\omega/m)m$$

1) The process $\gamma \to 2\gamma$ was considered in fourth order earlier by Skobov [5]. His result is not gauge invariant, owing to an error made in the calculation. A gauge-invariant expression was obtained by Sannikov [6], who investigated the inverse reaction, but the numerical coefficient in [6] is incorrect. We note that formula (1) is the first nonvanishing approximation in B/B_0 when $\omega < 2m$.

$\sin\theta\omega B\vec{N} = \vec{k}\times\vec{E}$; θ is the angle between $\vec{k}$ and $\vec{B}$; ω and $\vec{k}$ are the frequency and wave vector of the primary photon.

Let us consider reactions in which photons linearly polarized parallel ($\parallel$) and perpendicular ($\perp$) to the vector $\vec{N}$ take part. It follows from (2) that the reactions $\parallel \to \parallel + \parallel$, $\parallel \to \parallel + \perp$, $\perp \to \perp + \perp$, and $\perp \to \parallel + \parallel$ are forbidden in the considered lowest order in B/B_0. If we take further account of the vacuum-polarization effects, namely the double refraction in the magnetic field [7, 8], then the reaction $\perp \to \perp + \parallel$ is also forbidden. Indeed, the effect of refractive index for the "transverse" polarization is somewhat larger than for the "longitudinal" polarization (when $(\omega/m)(B/B_0) \ll 1$), and therefore the decay $\perp \to \perp + \parallel$ is forbidden by the energy-momentum conservation law. Thus, the only allowed reaction is the decay of the "parallel" photon into two "perpendicular" ones: $\parallel \to \perp + \perp$, which should lead to polarization of the initial unpolarized beam of photons in a plane passing through the direction of the magnetic field and the observation line. This conclusion agrees fully with the conclusion of [1].

Integrating (2) with respect to λ we obtain the reciprocal of the path length of the "parallel" photons:

$$l^{-1} \simeq 0.4\cdot 10^2 (B\sin\theta/B_0)^2 (\omega/m) \ (\text{cm}). \tag{3}$$

To estimate the magnitude of the effect under the proposed conditions of neutron stars we put $B \sim 0.1\ P_0$ and assume that $\sin\theta \sim 1$. Then for γ rays in the region $\omega \sim 0.1m$ (50 keV) the free path of the "parallel" photons is of the order of 25 cm. We note that according to the results of Adler et al. [1], at the same values of the parameters, the free path is larger by 10^{11} times.

Thus, our result shows that polarization of not only γ rays but also of x-rays (starting with several keV) is quite probable in pulsars.

[1] S.L. Adler, J.N. Bahcall, C.G. Callan, and M.N. Rosenbluth, Phys. Rev. 25, 1061 (1970).

[2] V.L. Ginzburg, V.V. Zheleznyakov, and V.V. Zaitsev, Usp. Fiz. Nauk 98, 201 (1969) [Sov. Phys.-Usp. 12, 378 (1969)].

[3] A. Hewish, Annual Review of Astronomy and Astrophys. 8, 265 (1970).

[4] V. Canuto, H.Y. Chiu, and C. Chiuderi, Nature 225, 47 (1970).

[5] V.G. Skobov, Zh. Eksp. Teor. Fiz. 35, 1315 (1958) [Sov. Phys.-JETP 8, 919 (1959)].

[6] S.S. Sannikov, ibid. 52, 1303 (1967) [25, 867 (1967)].

[7] R. Baier and P. Breitenlohner, Acta Phys. Austriaca 25, 212 (1967); Nuovo Cim. 47B, 117 (1967).

[8] N.B. Narozhnyi, Zh. Eksp. Teor. Fiz. 55, 714 (1967) [Sov. Phys.-JETP 28, 371 (1969)2.

E R R A T U M

The article by D.V. Gol'tsov and V.V. Skobelev, V. 13, No. 3, p. 122 contains an error. In calculating the matrix element in the Furry picture, no account was taken of the contribution due to the phase factor exp i (x, 7), which enters in the expression for the Green's function of an electron in a homogeneous field, G(x, y) = exp i (x, y)S(x - y). This contribution may turn out to be equal in magnitude and opposite in sign to the function S(x - y). Thus, in the approximation considered, the matrix element of the process vanishes. A similar error was made also in the papers cited in our article [5, 6]. A corrected version of the article has been submitted to Vestnik Moskovskogo Universiteta (Herald of the Moscow University).