$\sin\theta\omega B\vec{N} = \vec{k}\times\vec{E}$; θ is the angle between $\vec{k}$ and $\vec{B}$; ω and $\vec{k}$ are the frequency and wave vector of the primary photon.

Let us consider reactions in which photons linearly polarized parallel ($\parallel$) and perpendicular ($\perp$) to the vector $\vec{N}$ take part. It follows from (2) that the reactions $\parallel \to \parallel + \parallel$, $\parallel \to \parallel + \perp$, $\perp \to \perp + \perp$, and $\perp \to \parallel + \parallel$ are forbidden in the considered lowest order in B/B_0. If we take further account of the vacuum-polarization effects, namely the double refraction in the magnetic field [7, 8], then the reaction $\perp \to \perp + \parallel$ is also forbidden. Indeed, the effect of refractive index for the "transverse" polarization is somewhat larger than for the "longitudinal" polarization (when $(\omega/m)(B/B_0) \ll 1$), and therefore the decay $\perp \to \perp + \parallel$ is forbidden by the energy-momentum conservation law. Thus, the only allowed reaction is the decay of the "parallel" photon into two "perpendicular" ones: $\parallel \to \perp + \perp$, which should lead to polarization of the initial unpolarized beam of photons in a plane passing through the direction of the magnetic field and the observation line. This conclusion agrees fully with the conclusion of [1].

Integrating (2) with respect to λ we obtain the reciprocal of the path length of the "parallel" photons:

$$l^{-1} \simeq 0{,}4\cdot 10^2 (B\sin\theta/B_0)^2\,(\omega/m)\ (\text{cm}). \tag{3}$$

To estimate the magnitude of the effect under the proposed conditions of neutron stars we put $B \sim 0.1\ P_0$ and assume that $\sin\theta \sim 1$. Then for γ rays in the region $\omega \sim 0.1m$ (50 keV) the free path of the "parallel" photons is of the order of 25 cm. We note that according to the results of Adler et al. [1], at the same values of the parameters, the free path is larger by 10^{11} times.

Thus, our result shows that polarization of not only γ rays but also of x-rays (starting with several keV) is quite probable in pulsars.

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ALLOWANCE FOR THE COULOMB INTERACTION IN THE SCATTERING OF A FAST CHARGED PARTICLE BY A NUCLEUS

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1. In this paper, on the basis of the assumption that Glauber's formula for the scattering amplitude [1] holds when account is taken of only the strong interaction of the pion with the nucleons of nucleus A, we prove that to take into account the Coulomb interaction of the pion with protons it suffices to make the substitution $\delta^p(\vec{\rho}) \to \delta^p(\vec{\rho}) + \delta^c(\rho)$, where $\delta^p(\rho)$ and $\delta^c(\rho)$ are respectively the nuclear and Coulomb phases for the scattering of the pion by the

free proton, and $\vec{\rho}$ is the impact parameter. That is to say, when allowance is made for the Coulomb corrections, the amplitude of the πA scattering takes the form

$$F_A(s, \vec{\Delta}) = \frac{ik}{2\pi} \int e^{i\vec{\Delta}\vec{\rho}} \phi_f^*(r_1, \dots, r_A)(1 - \Gamma(\vec{\rho}, s_1, \dots, s_A)) \times \phi_i(r_1, \dots, r_A) d^2\rho \, d^3r_1 \dots d^3r_A , \tag{1}$$

where $\vec{\Delta}$ is the momentum transfer, k the momentum of the pion in the lab, $\sqrt{s}$ the energy,

$$\Gamma(\vec{\rho}, s_1, \dots, s_A) = 1 - e^{2i\delta(\vec{\rho}, s_1 \dots s_A)},$$

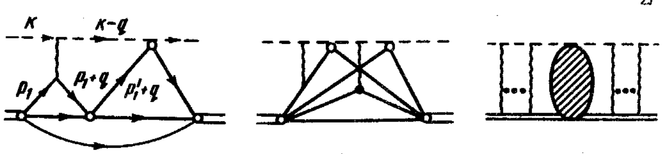
$$\delta(\vec{\rho}, s_1, \dots, s_A) = \sum_{l=1}^{z} (\delta^p(\vec{\rho} - s_l) + \delta^c(\vec{\rho} - s_l)) + \sum_{l=z+1}^{A} \delta^n(\vec{\rho} - s_l), \tag{2}$$

$$\delta^c(\vec{\rho}) = -\frac{\alpha}{2\pi} \int \frac{d^2 q_\perp}{q_\perp^2 + \lambda^2} \exp(-i\vec{\rho} q_\perp) \tag{3}$$

is the Coulomb phase of scattering of the pion by the free proton, and λ is the proton mass. A formula similar to (1) was obtained in [2] in the impulse approximation for A = 2 and in first order in α.

Additivity of the nuclear and Coulomb phases was assumed in [3 - 4], where Coulomb effects in the scattering of protons by nuclei were considered. Such an additivity actually does take place if the interaction between the incoming particle and the nucleons of the nucleus can be described with a superposition of two potentials, nuclear and the Coulomb. In the derivation of (1) we shall, however, not assume that at high energies a strong interaction can be described with the aid of the concept of the potential. On the other hand, when account is taken of the Coulomb interaction, it is impossible to employ a direct generalization of the available methods of deriving Glauber's formula on the basis of the diagram technique [5] or Watson's theory of multiple scattering [6], since the contribution of the rescattering of the nucleons in the nucleus between the electromagnetic and strong scattering acts of the incoming particles turns out to be not small, by virtue of the long-range character of the Coulomb potential [2].

2. In the diagram describing the cross scattering of the nucleons between the Coulomb and strong pion-scattering acts (Fig. a), the integral with respect to the 4-momentum of the virtual photon q converges at small values of q as a result of the propagators only (two nucleon, one pion, and one photon propagator). The main contribution to the integral is made by values of q such that all the denominators are close to zero. From the conditions $q^2 = 0$, $(k - q)^2 = m^2$, and $(p_1 + q)^2 = m_p^2$, with allowance for the fact that $p_{10} - m_p \sim p_1/2m \sim \varepsilon$ (ε is the binding energy per nucleon), it follows that in the diagrams with cross scattering an important role is played by the following values of q, viz., $q_0 \sim \varepsilon$, $q_z \sim q_0/v_\pi \sim \varepsilon$, and $q \sim m_\pi \varepsilon/k$ [2] (the z axis is directed along $\vec{k}$). Therefore in the integration with respect to $d^2q_\perp$ we shall distinguish between the region of "soft" photons $q_\perp^2 \leq q_{\perp,0}^2$ in the region of "hard" photons $q_\perp^2 \geq q_{\perp,0}^2$, and we choose $q_{\perp,0}$ such



a b c

that $(m_\pi \varepsilon/k)^2 << q^2_{\perp,0} << R^{-2}$, where R is the radius of the nucleus. Accordingly, the Coulomb phase $\delta^c(\vec{\rho})$ (see (3)) breaks up into terms $\delta^c(\vec{\rho}) = \delta^c_0(\vec{\rho}) + \delta^c_1(\vec{\rho})$, where $\delta^c_0(\vec{\rho})$ determines the contribution of the "hard" photon, and $\delta^c_1(\vec{\rho})$ that of the "soft" photon.

Let us sum now all the diagrams of the type of Fig. b, which take into account strong interaction in exchange of only "hard" photons. In considering diagrams of this type, we can use the known methods for deriving Glauber's formula [5 - 6], since the cross scattering of the nucleons is insignificant in this case. For the amplitude $F^0_A(s,\vec{\Delta})$, which takes into account the strong interaction and exchange of "hard" photons, we obtain in this case formula (1), in which $\delta^c(\vec{\rho})$ is replaced by $\delta^c_0(\vec{\rho})$. The complete amplitude, which takes into account also the contribution of the "soft" photons, will be represented in the form

$$F_A(s,\vec{\Delta}) = \sum_{m=0}^{\infty} F^m_A(s,\vec{\Delta}) + F^{soft}_A(s,\vec{\Delta}), \tag{4}$$

where $F^{soft}_A(s,\vec{\Delta})$ describes the contribution of only the "soft" photons, $F^m_A(s,\vec{\Delta})$ at $m \neq 0$ is the correction to $F^0_A(s,\vec{\Delta})$ due to exchange of m "soft" photons. The contribution of the "soft" photons with momenta q_0, $q_z << k$, $q^2_\perp << R^{-2}$ for $\vec{\Delta}^2 << s$ is described by pole diagrams of the type of Fig. c [7]. Using a direct generalization of formula (16) of [7], we obtain the following expressions:

$$F^{soft}_A \text{ и } F^m_A:\ F^{soft}_A(s,\vec{\Delta}) = \frac{ik}{2\pi}\int e^{i\vec{\Delta}\vec{\rho}}(1 - e^{2i\delta^c_1(\vec{\rho})})d^2\rho,$$

$$F^m_A(s,\vec{\Delta}) = \frac{1}{m!}\left(-\frac{i\alpha z}{\pi}\right)^m \int \prod_{l=1}^{m} \frac{d^2\vec{q}^{\,l}_\perp\,\theta(q^2_{\perp,0} - q^{l\,2}_\perp)}{q^{l\,2}_\perp + \lambda^2} F^0_A(s,\vec{\Delta} - \sum_{l=1}^{m} q^l_\perp).$$

If we change over now to the ρ-representation in (4) and in F^m_A, then we obtain the following expression for the total "shaping function" Γ:

$$\Gamma(\vec{\rho}, s_1,\ldots, s_A) = \Gamma^{soft} + \sum_m \Gamma^m = 1 - e^{2i\delta^c_1(\vec{\rho})}$$
$$+ e^{2i\delta^c_1(\vec{\rho})}\left\{1 - \exp\left(\sum_{l=1}^{z}(\delta^p(\vec{\rho} - s_l) + \delta^c_0(\vec{\rho} - s_l)) + \sum_{l=z+1}^{A}\delta^n(\vec{\rho} - s_l)\right)\right\} \tag{5}$$

Since by virtue of the condition $q_{\perp,0}R << 1$ the phase $\delta^c_1(\vec{\rho})$ remains practically unchanged by the substitution $\vec{\rho} \to \vec{\rho} - \vec{s}_l$, we have verified that representation (1) for the scattering amplitude actually holds true.

3. Greatest interest attaches apparently to the application of formula (1) to the calculation of the difference of the differential and total cross sections for the scattering of π^- and π^+ mesons by nuclei with isospin zero. In this case we are rid of the uncertainties contained in the terms of zeroth order in α, which make no contribution to such differences. We consider here the difference of the total cross sections of π^- and π^+, σ^- and σ^+ in forst order in α. Calculating $\mathrm{Im}F_a(s, 0)$ and using the optical theorem, we obtain the following expression for r:

$$r = 2\frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+} = 2\alpha z \eta C_o^{-1}\left(C_1 \ln\frac{1}{R^2 t_{min}} + C_2 \ln\frac{1}{R_o^2 t_{min}} + C_3\right). \tag{6}$$

Here

$$C_o = \frac{\sigma_{\pi A}^{tot}}{2\pi R^2},\quad \eta\frac{(C_1 + C_o)}{C_o} = \frac{\mathrm{Re}\,F_A(s,0)}{\mathrm{Im}\,F_A(s,0)},\quad F_{\pi\pm p}(-\vec{\Delta}) = \frac{k\sigma_{\pi p}^{tot}}{4\pi}(i + \eta^\pm)e^{R_o^2 t},$$

$\eta = (\eta^+ + \eta^-)/2$ and t_{min} is the momentum-transfer resoltuion in the experiment. The ratio $C_2/C_1 \sim R_0^2/R^2 \sim 1/A^{2/3}$, i.e., it is small when $A >> 1$. If we estimate the coefficients C_i under the same assumptions as are made in the analysis of the elastic cross sections of πA scattering with the aid of Glauber's formula [3] (factorization of the square of the modulus of the nuclear wave function, a definite choice of the single-particle density), then we obtain the values $C_1/C_0 = 0.72$ (for He^4), 0.73 (C^{12}), 0.67 (O^{16}), 0.52 (Ca^{40}); $C_2/C_0 = 0.17$ (He^4), 0.03 (C^{12}), 0.012 (O^{16}), <0.005 (Ca^{40}); $C_3/C_1 < 0.1$ for all the cases in question. For the nuclei He^4, C^{12} and O^{16} we have chosen the single-particle density in the form $\rho(r) = (\pi R)^{-3/2}\exp(-r^2/R^2)$, where R = 1.3, 1.6, and 1.7 F for He^4, C^{12}, and O^{16}, and for Ca^{40} we used the model of a nucleus with a sharp boundary, $R = 1.2 \times A^{1/3}$F. In addition, we have assumed $\eta^+ = \eta^- = -0.2$ and $R_0^2 = 0.2\ (\mathrm{GeV}/c)^2$, choosing the pion energy in the region of 5 GeV. It should be noted that the ratio C_1/C_2, which determines in the main the value of r, depends little on the model for the cases in question. We thus obtain $C_1/C_0 \approx 0.65$ for C^{12} and O^{16} in the nuclear model with a sharp boundary.

The main uncertainties in formulas (1) and (6) are due to the following: a) neglect of the contribution of the inelasticities in the intermediate states, which can become appreciable when $k > m_\rho^2 R$ [8]; b) non-asymptotic terms connected with the error introduced when the propagator of the particles is replaced by a δ-function (see (2)) (relative contribution of these terms to $r \sim (Rk\eta)^{-1}$); c) the uncertainty in the radiative corrections to the πN amplitude ($\sim R_0^2/R^2$); d) the Coulomb corrections to the wave function of the nucleus ($\sim(\eta^- - \eta^+)/2\eta$). If the experimental resolution is such that $R^2 t_{min} << 1$, then the relative contribution of the errors to r weakens by a factor $\ln[1/R^2 t_{min})]$.

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