POSSIBILITY OF SHORTENING LIGHT PULSES IN ALKALI-METAL VAPOR

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We discuss here the possibility of self-shortening of the time of a light pulse during the course of its propagation in a nonlinear reactive medium with strong dependence of the group velocity on the frequency. Such a possibility was already noted in the literature [1, 2] and is due to the action of the following two effects. First, when the light pulse passes through the nonlinear reactive medium there occurs a frequency self-modulation (see [3]), and in the case of a sufficiently strong nonlinearity the resultant broadening of the spectrum may turn out to be large: $\delta\omega_1 >> \tau_0^{-1}$, where τ_0 is the duration of the initial pulse. Second, the frequency-modulated pulse with a spectrum width $\delta\omega_1$ can be shortened in time, to a minimum duration $\tau_1 \sim (\delta\omega_1)^{-1} << \tau_0$) with the aid of a linear system in which the time delay of the arrival depends in the required manner on the frequency [4]. We emphasize that a system of diffraction gratings was used as the linear dispersive system in the reported investigations.

This raises naturally the question whether it is possible to find media in which both phenomena, nonlinear broadening of the spectrum and dispersion shortening of the time, could be simultaneously realized. This question is the subject of the present communication.

We present first all the formulas needed for the estimates. During the propagation of a light pulse in a medium with reactive nonlinearity, the light field acquires an additional phase (frequency) modulation

$$\phi(z,t) = \frac{\omega_0}{c} z n_2 \left| E_0\left(t - \frac{z}{v}\right)\right|^2;$$

$$\delta\omega_{\rm inst} = -\frac{\partial\phi}{\partial t} \sim z\frac{\omega_0}{c} n_2 |E_0|^2 \tau_0^{-1}, \tag{1}$$

where $n = n_0 + n_2|E|^2$ is the refractive index, which depends on the intensity, $\delta\omega_{\rm inst}$ is the addition to the instantaneous frequency at the instant of time t in the cross section z, and ω_0 is the central frequency of the light pulse at the entrance into the medium (at z = 0). Let $n_2 > 0$; then the frequency self-modulation causes the lower frequencies to be located in the leading part of the pulse, where $|E(t)|$ increases. If $(dv_{gr}/d\omega) > 0$, then the higher frequencies will overtake the lower ones during the course of the propagation, and the pulse will contract in time. Similar arguments can be applied also to the case $n_2 < 0$. It turns out finally that the condition for the time self-shortening can be written in one of the following forms:

$$n_2 \frac{dv_{gr}}{d\omega} = -n_2 v^2 \frac{d^2 k}{d\omega^2} \equiv -n_2 \frac{v^2}{c^2}\frac{\lambda^3}{2\pi}\frac{d^2 n_0}{d\lambda^2} > 0. \tag{2}$$

The length over which there is a noticeable effect of dispersion change in the shape of a pulse with duration τ_0 and spectral width $\delta\omega >> \tau_0^{-1}$ is

$$z \sim \tau_0 v^2/\delta v \sim \tau_0 v^2 \left[\frac{dv}{d\omega}\delta\omega\right]^{-1}.$$

If we substitute here the order of magnitude of $\delta\omega$ from (1), then we obtain the length z_1 over which the self-contraction effect becomes noticeably manifest:

$$z_1 \propto c r_0 \left\{ - n_2 |E_0|^2 c\omega \frac{d^2 k}{d\omega^2} \right\}^{-1/2}. \tag{3}$$

For most media in the visible region $n_2 > 0$ and $(d^2 n_0/d\lambda^2) > 0$; therefore the sign of the effect corresponds to self-lengthening of the pulse. But even this latter effect could appear in ordinary media only at exceptionally large lengths z_1. In fact, assuming $P = 100$ MW/cm^2, $\tau_0 \approx 10^{-11}$ sec, $n_2 \approx 10^{-13}$ cgs eus, and $\lambda^2(d^2n_0/d\lambda^2) \approx 1$, we get $z \sim 10^3$ cm.

We wish to call attention to the fact that media suitable for the realization of self-shortening are rarefied vapors of alkali metals. For such media, owing to the smallness of the width γ of the atomic lines, the radiation can be chosen to be quite close to the frequency of the resonant transition of the atom, and by the same token it is possible to realize a large value of both the nonlinearity[1] and of the dispersion.

When the field interacts with the atoms near resonance, we have

$$n_0(\omega) = 1 + \frac{2\pi N p_{12}^2}{\hbar(\omega_{12} - \omega)}; \quad n_2 = (n_0 - 1)\frac{p_{12}^2}{4\hbar^2(\omega_{12} - \omega)^2}. \tag{4}$$

It is easy to see that near strong resonance (but at $|\omega_{12} = \omega| \gg \gamma$) we have

$$\frac{dv_{gr}}{d\omega} > 0, \; n_2 > 0 \text{ for } \omega > \omega_{12} \text{ and } \frac{dv_{gr}}{d\omega} < 0, \; n_2 < 0 \text{ for } \omega < \omega_{12}.$$

Therefore the self-shortening condition (2) is automatically satisfied both above and below the resonance.

We present numerical estimates of the quantities in (3) and (4), making use of the conditions of experiments [5] with potassium vapor. For the resonant transition $4S_{1/2} - 4P_{3/2}$ in potassium, the square of the matrix element of the z-component of the dipole moment is $p_{12}^2 = 3.4 \times 10^{-35}$ cgs esu. Let $N = 2 \times 10^{16}$ cm^{-3} and $|\omega_{12} - \omega|/2\pi c = 12$ cm^{-1}. In this case the refractive index is $n_0 = 1 + 2 \times 10^{-3}$, $v_{gr} = c/3$, and the dimensionless dispersion is $c\omega(d^2k/d\omega^2) = 4 \times 10^3$. Assuming a power density $P \approx 20$ MNb/cm^2, we get $n_2|E_0|^2 \approx 0.5 \times 10^{-3}$,[2] and then $z_1 \approx 0.7c\tau_0$. Thus, $z_1 \sim 2$ cm for $\tau_0 \approx 10^{-10}$ sec and $z_1 \approx 0.2$ cm for $\tau_0 \approx 10^{-11}$ sec. Such pulse durations and interaction lengths can be realized experimentally.

Let us estimate now the role of the absorption of light. The absorption coefficient (in cm^{-1}) near resonance is $\alpha = (n_0 - 1)\gamma\omega/|\omega_{12} - \omega|c$ (we assume that $\gamma \ll |\omega_{12} - \omega|$). Then, according to (3), and (4), we have

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[1]Broadening of the resonant-radiation spectrum in alkali-metal vapors was observed in [5], where the statement was made that this broadening is due to self-modulation.

[2]We note that at these values of the power density we have $n_2|E_0|^2 \approx 0.25(n_0 - 1)$. This means that the system is close to saturation of nonlinearity, which in turn can greatly influence the character of the final stage of the self-shortening.

$$\alpha z_1 \approx \gamma \tau_0 \left\{ n_2 |E_0|^2 / (n_0 - 1) \right\}^{-1/2} \quad (5)$$

In the case of broadening due to resonant interactions, the line width γ (in sec^{-1}) is (see [6] Sec. 40) $\gamma \simeq 2p_{12}^2N/h$. For the discussed numerical example, $\gamma \simeq 2 \times 10^9$ sec^{-1} and $n_2|E_0|^2/(n_0 - 1) \simeq 0.25$. Then $\alpha z_1 \simeq 0.4$ for $\tau_0 = 10^{-10}$ sec and $\alpha z_1 \simeq 0.04$ for $\tau_0 = 10^{-11}$ sec, and consequently the beam attenuation due to absorption is small over the self-shortening length.

To find the final duration of the self-shortened pulse it is necessary to take accurate account of the saturation of the nonlinearity (which in the case of a resonant transition leads to a decrease of the dispersion). We present here a formula for the duration τ of the self-shortening pulse under the assumption that the dispersion is constant in the entire frequency interval $\delta\omega \sim 1/\tau$ and does not depend on the field amplitude:

$$\tau \sim \frac{1}{\omega_0} \left\{ c\,\omega_0 \frac{d^2k}{d\omega^2} \Big/ \delta n_{\text{nonlin}} \right\}^{-1/2}. \quad (6)$$

If we put $\delta n_{\text{nonlin}} \simeq (n_0 - 1)$, then Eq. (6) yields $\tau \sim 10^{-12}$ sec for the numerical example considered above. If we write, on the other hand, $\delta n_{\text{nonlin}} = n_2|E|^2$ and use (4), then we get from (6)

$$\tau \sim \hbar / p_{12} E. \quad (7)$$

Relation (7) between τ and E has approximately the same form as in the known case of 2π pulses (see [7]).

However, the possibility of formation of a 2π pulse as a result of the described effect of self-shortening calls for a special analysis.

Resuming, we can state that there is a possibility of shortening the duration of a light pulse from $\tau_0 \sim 10^{-10} - 10^{-11}$ sec to $\tau \sim 10^{-12}$ sec by interaction of resonant radiation with potassium vapor.

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