(corresponding to an energy $T_0 = \delta T_{max}$)

$$\theta_0 \approx \sqrt{(2m_e/T_0)(1-\delta)}. \tag{10}$$

In the model of vector dominance with current mixing we have $\langle r^2 \rangle = 0.76\times10^{-27}$ cm^2 [6], i.e., R_K = 0.275 F.[3] If we assume this value, then at an energy E_K = 20 GeV and δ = 1/10 (γ = 40, T_{max} = 1.6 GeV, $t_{max}R_K^2 = 3\times10^{-3}$, θ_0 = 4.3°) the integral regeneration cross section calculated by formula (9) equals 1.72×10^{-35} cm^2. To estimate the number of events we assume that a liquid-hydrogen target of length L = 300 cm is used, and that the intensity of the K_L^0-meson beam is I = 10^5 particles/sec. We can then expect that after 100 hours of continuous accelerator operation there will be registered approximately eight acts of the regeneration $K_L^0 + e \to K_S^0 + e$. At an energy E_K = 200 GeV and at the same values of R_K, δ, L, and I (corresponding to γ = 400, T_{max} = 90 GeV, $t_{max}R_K^2$ = 0.17, and θ_0 = 0.57°), the regeneration cross section is already equal to 1×10^{-33} cm^2, and the number of expected events increases to 5 per hour. We note that by choosing targets of dense substances with heavy nuclei it is possible in principle to increase the rate of statistics accumulation by 2 - 3 times (the maximum number of events, with allowance for absorption of the K_L^0 mesons in the medium, is proportional to $(Z/A)A^{1/3}$). In this case, however, the radiation length decreases substantially, and difficulties may arise in the registration of the recoil electrons. The question of the optimal conditions for performing the experiments calls for a special study.

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1) We do not consider here the effect of coherent (refraction) regeneration $K_L^0 \to K_S^0$ on electrons, which was indicated by Ya. B. Zel'dovich [1]. This effect was investigated experimentally by the authors of [3].

2) It can be verified that CPT invariance holds, and Eqs. (4) remain in force also when account is taken of CP-parity nonconservation in neutral K-meson decays [5].

3) According to [3], $R_K = (0.2^{+0.22}_{-0.2})$ F.

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RESONANT PAIR PRODUCTION IN A STRONG ELECTRIC FIELD

V. S. Popov
Institute of Theoretical and Experimental Physics
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Pair production form vacuum by an external electromagnetic field has been evoking considerable interest in recent years (see [1 - 4] and the references cited therein). Experimental observation of this process would mean a verification of one of the important predictions of quantum electrodynamics [5]. Such an experiment will apparently be made possible by progress in laser technology. The e^+e^- pairs are produced in this case by a field F(t) that depends periodically on the time. In this article we wish to call attention to the fact that pair production has in this case certain peculiarities connected with the periodicity of the field.

Let $F(t) = F\phi(\tau)$ be the alternating electric field, F its amplitude, $\tau = \omega t$, $T = 2\pi/\omega$ the period,

$$\phi(\tau + 2\pi) = -\phi(\tau + \pi) = \phi(\tau) \tag{1}$$

and $|\phi(\tau)| \leq \phi(0) = 1$ (i.e., the field is maximal at the instants $t = k\pi/\omega$, k = 0, ±1, ±2, ...). If the conditions

$$\omega \ll m, \; F \ll F_0 = m^2c^3/e\hbar \tag{2}$$

are satisfied (henceforth $\hbar = c = m = 1$), it is convenient to solve the problem by the imaginary-time method [3]. The amplitude of pair production from vacuum is determined by the complex branch points $t = t_k$ in the upper half-plane of the "time" t. They are determined from the condition $p^2(t_k) + m^2 = 0$, where $\vec{p}(t)$ is the momentum of the classical particle. For a periodic field, all these points are located at equal distances from the real axis (see the figure):

$$r_k = \omega t_k = k\pi + (-1)^k r' + i r'' . \tag{3}$$

We denote by $\vec{p} = (p_\parallel, p_\perp)$ the momentum of the electron at the instant $t = 0$, when e^- and e^+ cross the continuum limits and emerge from under the barrier. Expanding in powers of $p \ll 1$, we have

$$\begin{aligned} r' &= \gamma\psi(\gamma)p_\parallel + O(p^3) \\ r'' &= \int_0^\gamma \psi(x)\,dx + \frac{1}{2}\left[\gamma\psi(\gamma)p_\perp^2 - \gamma^2\psi'(\gamma)p_\parallel^2\right] + \ldots \end{aligned} \tag{4}$$

Here $\gamma = m\omega/eF$ is the adiabaticity parameter for under-the-barrier motion, $\psi(x)$ is a function determined by the form of the applied field $\phi(\tau)$. The procedure from changing from ϕ to ψ is described in [4]. For example, for a monochromatic field we have $\phi = \cos\tau$ and $\psi(x) = (1 + x^2)^{-1/2}$; for other examples see [4].

From the point of view of Dirac's theory, the production of e^+e^- pairs by an electric field F(t) can be described as a transition of an electron with negative energy under a barrier, through a gap between the lower and upper continua. In the "imaginary time" plane, this corresponds to a contour circuiting around one of the branch points of the action function $S = \int L dt$, see [3]. Let A_k be the amplitude of the transition corresponding to the circuiting of the k-th branch point (see the figure). In a periodic field, the amplitudes A_k add up coherently, leading to a resonance effect. It follows from (3) that

$$A_{2k+1}/A_{2k} = \sigma \exp\{i(\phi - \beta)\}, \tag{5}$$

where

$$\phi = \int_0^T \epsilon(t)\,dt = \frac{\pi m}{\omega}(\Delta + \Delta_1 p_\perp^2 + \Delta_2 p_\parallel^2), \tag{6}$$

$$\Delta = \frac{2}{\pi}\int_0^\pi d\tau \left\{1 + \frac{1}{\gamma^2}\left(\int_0^\tau \phi(x)\,dx\right)^2\right\}^{1/2}, \quad \beta = \frac{2}{\omega}\int_0^{r'} \epsilon(t)\,dt = 2\gamma\psi(\gamma)p_\parallel/\omega \tag{6'}$$

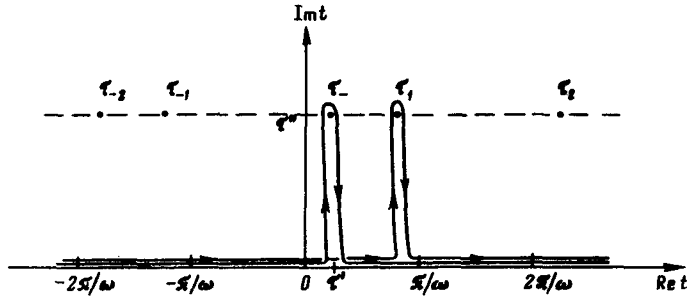
Δ_1 and Δ_2 are expressed in terms of Δ, and $\sigma = (-1)^{2s}$ is a signature factor having opposite signs for bosons and fermions ($s = 0, 1/2$). The appearance of σ is due to the fact that the amplitudes A_k and A_{k+1} correspond to opposite directions of the external field, and reversal of the sign of $\vec{F}$ (at a given $\vec{p}$) is equivalent to permutation of the charged particles. Summing the contributions A_k from 2N branch points, we go over as $N \to \infty$ to the probability of pair production per unit time

$$w = \lim_{N\to\infty} \frac{\omega}{2\pi N}\left|(1 + \sigma e^{i(\phi-\beta)})\sum_{k=0}^{N-1} e^{2ik\phi} A_0\right|^2 = \sum_{n=\nu}^{\infty} w_n, \quad \nu = \Delta/\omega \tag{7}$$

$$w_n = (2s+1)\frac{\omega^2}{\pi}\int \frac{d^3p}{(2\pi)^3}\left[1 + \sigma(-1)^n \cos\beta\right] w(p)\,\delta(\Delta + \Delta_1 p_\perp^2 + \Delta_2 p_\parallel^2 - n\omega) \tag{8}$$

Here w_n is the probability of pair production following absorption of n quanta, Δ is the effective width of the gap between the continua (the increase of Δ in comparison with $\Delta_0 = 2m$ for a free electron is connected with the energy of the oscillation in the external field). The factor $w(\vec{p})$ represents the contribution of an individual branch point and has been calculated earlier [3, 4]. It takes the form of a Gaussian distribution with respect to $p_\parallel$ and $p_\perp$. The integrand in (8) determined the momentum spectrum of the produced particles. The integration with respect to d^3p can be carried through to conclusion so that w_n can be expressed in terms of tabulated functions. Owing to the factor σ, the probabilities w_n depend on the type of statistics of the produced particles.

The influence of the statistics is most noticeable at p = 0, i.e., for particles emitted in a direction perpendicular to the electric field. In this case the probabilities w_n are proportional to $1 + \sigma(-1)^n$, i.e., the boson production occurs o,curs only at even harmonics n = 2k, while fermions are produced at odd harmonics. This selection rule is unique to a periodic field and is due to interference of the amplitudes A_k. The interference effect is 1 in the range of angles 90° ± θ, where $\theta \sim [\gamma\psi(\gamma)]^{-1}(\omega/m)^{1/2}$ at $\gamma >> 1$. In particular, $\theta \sim (\omega/m)^{1/2}$ for a sinusoidal field. This effect is appreciably expressed in the total probability w_n.



With the aid of (7) and (8) we can trace the continuous transition from γ = 0 (constant field) to γ >> 1 (in this case the field has time to reverse polarity many times during the time of flight of the electron through the barrier). Formula (7) at γ = 0 agrees with Schwinger's result [5] for a constant electric field, and goes over at γ >> 1 into perturbation theory. In this case the w_n decrease exponentially with increasing n. Thus for φ = cos τ we have $w_{n+1}/w_n \sim (e/4\gamma^2)^2 << 1$.

These results are valid at times t such that $\omega^{-1} << t << w^{-1}$. The time t = NT of action of the field extends over many periods and resonance has time to be produced, while the total transition probability is still small, W(t) = wt << 1. It is only under these conditions that the transition probability per unit time has any meaning. At F ≳ F and wt > 1 it is necessary to have an exact solution of the Dirac equation. Let n(t) be the average number of pairs produced within the time t. For fermions, n(t) oscillates periodically, and for bosons it can increase exponentially, $n(t) \sim e^{\mu t}$. It can be shown that this behavior of n(t), which was derived in [6] for a field φ(τ) = sin τ, holds true for any periodic field and is connected with the group properties of the pair-production problem (the symmetry group is SU(2) for fermions and SU(1, 1) for bosons).

We present numerical estimates. For electron-positron pairs $F_0 = 1.3\times10^{16}$ V/cm. At $F > F = \omega F_0/m$, we have γ < 1, so that the adiabatic approximation can be used ($\hbar\omega$ = 1.8 eV and $F_1 = 5\times10^{10}$ V/cm for a ruby laser). Assuming that the maximum field intensity F is attained by focusing in a volume $v \sim \lambda^3$ (the diffraction limit) and that the pulse lasts N periods, we have for the number of n of the pairs produced by the field φ(τ) = cos τ:

$$n = N\left(\frac{m}{\omega}\right)^4\left(\frac{2F}{F_0}\right)^{5/2}\exp(-\pi F_0/F),$$

At $N = 10^5$ (pulse duration $t = NT \sim 10^{-11}$ sec) the field required to produce one e^+e^- pair is $F_{min} = 7\times10^{14}$ V/cm. On the other hand if v = 1 cm³ and t = 1 sec, then $F_{min} \sim 3\times10^{14}$ V/cm. At $F > F_{min}$ the number of pairs increases so strongly with increasing intensity F, that one can speak of laser breakdown of the vacuum. At F = πF we can assume that one e^+e^- pair is produced in a Compton unit of invariant volume $vt = \hbar^4/m^4c^5 \sim 10^{-52}$ cm³sec. This is the region ($F \gtrsim F_0$) of phenomena outside the framework of the linear regime W(t) = wt, which have been briefly discussed above.

The requirements on F_{min} will become somewhat less stringent if x-ray or γ-ray lasers are developed. In this case the laser breakdown of vacuum would begin in the anti-adiabatic region γ >> 1 and $n \sim m^4vt(F/F_1)^{2K}$, where K = 2m/ω. Thus, at $\hbar\omega$ = 20 keV we have K = 50, $F_{min} \sim 1.5\times10^{14}$ V/cm. The periodicity of the external field F(t) leads to a δ function in (8), thus pointing to a multiquantum character of the pair emission from the vacuum. This can be verified experimentally by measuring the summary energy of e^- and e^+.

In the boson case, the lightest charged particles are π^+ and π^-, for which $F_0 = 10^{21}$ V/cm (F_0 increases in proportion to m^2).

In conclusion, we note the formal analogy between the problem of the production of fermion pairs from vacuum and the problem of resonant excitation of atomic levels in the field of a strong optical wave, recently considered by D. F. Zaretskii and V. P. Krainov. The reason for this analogy is that the quasienergy of a two-level system in the field (1) is given by $E_{1,2}(t) = \pm(\varepsilon/2)\sqrt{1 + \beta^2\phi^2(\tau)}$, which recalls the relativistic dispersion law $E(t) = \pm\sqrt{m^2 + p^2(t)}$ in a field $\phi_1(\tau) = \phi'(\tau)$. If excitation of multilevel atomic systems is considered, the spectrum of the quasienergies $E_i(t)$ has a more complicated form, and this analogy no longer holds.

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MECHANISMS OF INELASTIC ION-ATOM COLLISIONS

Z. Z. Latypov and A. A. Shaporenko
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
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We investigate the effective charge-exchange and ionization cross sections in the systems Li^+-He, Li^+-Ne, H^+-He, and H^+-Ne at low collision energies. It is shown that the cross sections of the inelastic processes depend strongly on the relative placements of the curves of the potential energies of the ground and excited states of the quasimolecules produced in the collisions. Hypotheses are advanced concerning the probabilities of the mechanisms responsible for inelastic processes in these systems.

We present here the results of an investigation of the mechanisms of inelastic ion-atom collisions at low energies (100 - 1000 eV).

To obtain information on the dependence of the cross sections on the inelastic-collision mechanisms we have investigated the charge-exchange and ionization processes in two types of ion-atom systems: (a) H^+-He, H^+-Ne and (b) Li^+-He, Li^+-Ne. From an examination of the adiabatic correlation diagrams and the existing theoretical data on the potential energies of certain systems [1 - 3] it follows that the relative positions of the potential energy curve of the ground state and of the excited states are different in cases (a) and (b). It is known that in case (b) the ground-state potential-energy curves interact with the intermediate-quasimolecule excited-state curves either because of the Σ-Σ radial coupling that appears near the pseudo-crossing, or owing to the Σ-Π rotational coupling. In case (a) the ground states are not coupled with the excited states. One can therefore expect the mechanisms of the inelastic processes to be different in these two cases.

Using the experimental methods developed by us [4] for the registration of the secondary ions and free electrons produced when an ion beam passes through a gas, we measured the cross sections of the following processes:

$$H^+ + He \to H + He^+ \quad (1a)$$
$$H^+ + He \to H^+ + He^+ + e \quad (1b)$$
$$H^+ + Ne \to H + Ne^+ \quad (2a)$$
$$H^+ + Ne \to H^+ + Ne^+ + e \quad (2b)$$
$$Li^+ + He \to Li + He^+ \quad (3a)$$
$$Li^+ + He \to Li^+ + He^+ + e \quad (3b)$$
$$Li^+ + Ne \to Li + Ne^+ \quad (4a)$$
$$Li^+ + Ne \to Li^+ + Ne^+ + e \quad (4b)$$