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PION CONDENSATION IN NEUTRON STARS

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Submitted 18 July 1973
ZhETF Pis. Red. 18, No. 7, 443 — 447 (5 October 1973)

An expression is obtained for the energy density of nuclear matter in the presence of π-condensate of arbitrary density. This expression differs strongly from that obtained in [5]. It is shown that the second-order phase transition proposed in [5], with formation of π-condensate, is not realized. It is proved that the proposed model is also stable with respect to a first-order phase transition.

1. Introduction

It was shown in [1] that a phase transition with formation of a -condensate is produced in a sufficiently dense nucleon medium. A method of finding the spectrum of the pions in nuclear matter was developed in [2], where it was shown that an electrically-neutral condensate of π^0, π^+, and π^- mesons is produced at N = Z and at a nucleon density $n_c < n_0$ (n_0 is the nuclear density). The same method was used in [3] and [4] to consider the case Z << N (neutron stars). A condensate of π^0 mesons is produced at a density $n \simeq 0.6n_0$ and an electrically-neutral $\pi^+\pi^-$ condensate is produced at approximately the same density. The π^--meson condensate proposed in [5] is not produced. We show here where the authors of [5] have made their mistake. It is obvious that the expression obtained in [5] for the energy density is incorrect, since this expression shows a non-analytic dependence on the interaction constant even at a density lower than critical! The spectra of the π^0, π^+, and π^- mesons for Z << N are given in [3] and [4]. The condition for the second-order phase transition is the appearance of instability in these spectra. To assess the feasibility of a first-order phase transition with immediate formation of a condensate of finite density it is necessary to find the energy of the system at an arbitrary density of the condensate and to compare this energy with that of a system in which no phase transition has occurred, or (if $n > n_c$) with the energy of a system in which the indicated second-order phase transitions have taken place.

We shall determine the system energy in a model consisting of nucleons and a π^--meson condensate with wave vector $\vec{k}$. An attempt to solve this problem was made in [5]. However, in addition to a crucial calculation error, which will be dealt with later, the average-field method used in [5], even if correctly applied, cannot yield quantitative results and should be replaced by a more accurate method in which the effective Lagrangian of the pions is determined. It will be shown that the model considered here is stable with respect to a first-order phase transition. A second-order phase transition occurs in this model under the condition $\omega(k) < \varepsilon_F$ ($\omega(k)$ is the energy of the ions in the medium), which is not realized in the real case [3]. We note that all the objections advanced so far against the method developed in [2] turned out to be untenable (see [4]).

2. The Average-Field Method

For the sake of brevity, we use a symbolic notation, omitting the momentum and spin indices. The Hamiltonian H can then be expressed in the form ($\hbar = c = m_\pi = 1$)

$$H = \Sigma E^{(n)} n^+ n + \Sigma E^{(p)} p_0^+ p_0 + \omega_0 a^+ a + iM(np_0^+ a^+ - n^+ p_0 a), \quad (1)$$

where $M = fk/\sqrt{\omega_0}$; n, p_0, and a are the operators of the neutron, proton, and pion fields, and

$\omega_0^2 = 1 + k^2$.

The operators a and a^+ are replaced by the classical quantities, and we consider in lieu of (1) the following Hamiltonian (the "average-field method" [5])

$$H = \Sigma E^{(n)} n^+ n + \Sigma (E^{(p)} + \mu) p^+ p + iM\sqrt{\nu}(np^+ - n^+ p) + (\omega_0 - \mu)\nu. \tag{2}$$

with the additional electroneutrality condition $\Sigma\langle p^+ p\rangle = \nu$.

We note that the Hamiltonian (2) can be obtained from (1) by making the substitutions $a = \sqrt{\nu}\exp(i\omega t)$ and $a^+ = \sqrt{\nu}\exp(i\omega t)$. By shifting the proton energy by an amount $\mu = \omega$, i.e., by introducing $\hat{p} = \hat{p}_0 \exp(i\omega t)$ and $\hat{p}^+ = \hat{p}_0^+ \exp(-i\omega t)$ we obtain the time-independent Hamiltonian (2). It is thus clear beforehand that μ in (2) coincides with the new pion frequency $\omega(k)$, and in the case of Hamiltonian (1) it can be obtained by calculating the pion polarization operator. We shall see that the same result is obtained when the Hamiltonian (2) is used correctly. Since only two states take part in (2), namely a neutron with energy $E^{(n)}(p)$ and a proton with energy $E^{(p)}(\vec{p} + \vec{k})$ ($\vec{k}$ is the condensate unit wave vector), it follows that the new energies of the nucleons are obtained from a simple secular equation. Neglecting throughout the difference between the kinetic energies $E^{(n)}(p) - E^{(p)}(\vec{p} + \vec{k})$ (as shown in [5], the resultant error is small), we obtain

$$\tilde{E}^{(n)} = E^{(n)} - \frac{\mu}{2}\left(- \sqrt{1 + \frac{4M^2\nu}{\mu^2}}\right); \quad \tilde{E}^{(p)} = E^{(p)} + \frac{\mu}{2}\left(1 + \sqrt{1 + \frac{4M^2\nu}{\mu^2}}\right). \tag{3}$$

For the total energy of the system we have

$$E = \sum_{p < p_F^n} \tilde{E}^{(n)}(p) + \sum_{p < p_F^p} \tilde{E}^{(p)}(p) + (\omega_0 - \mu)\nu, \tag{4}$$

where

$$(p_F^n)^3/3\pi^2 = n - \tilde{\nu}; \quad (p_F^p)^3/3\pi^2 = \tilde{\nu}$$

$n - \tilde{\nu}$ is the number of "new" neutrons and $\tilde{\nu}$ is the number of "new" protons. From (3) and (4) we obtain

$$E = E_0[(1 - \tilde{x})^{5/3} + \tilde{x}^{5/3}] + (n - 2\nu)\frac{\mu}{2} + \frac{n - 2\tilde{\nu}}{2}\mu\sqrt{1 + \frac{4M^2\nu}{\mu^2}}, \tag{5}$$

where $\tilde{x} = \tilde{\nu}/n$ and $E_0 = (3/5)n\varepsilon_F$. Expression (5) contains two parameters, μ and ν. Minimizing E with respect to μ, we obtain

$$(n - 2\nu)\gamma = n - 2\tilde{\nu}; \quad \gamma = \sqrt{1 + \frac{4M^2\nu}{\mu^2}} \tag{6}$$

is the density ratio of the old (ν) and new ($\tilde{\nu}$) protons. It is easily verified that the same ratio is obtained from the condition $\Sigma\langle p^+ p\rangle = \nu$. Minimizing with respect to ν, we get

$$\mu = \omega_0 - \frac{(n - 2\tilde{\nu}) f^2 k^2}{\omega_0 \mu \gamma}. \tag{7}$$

It is readily seen that (7) is an equation for the new frequency of the pions in the Hamiltonian (1), in accord with the relation $\mu = \omega(k)$. The error made in [5] was that after finding the minimum of the quantity $E + \mu\nu$ (which should not be minimal), the authors put $\tilde{\nu} = 0$. It is easy to see that (6) with $\tilde{\nu} = 0$ does indeed lead to the incorrect expression given for μ in [5].

After eliminating ν and μ with the aid of (6) and (7) we obtain an expression for the energy as a function of the number of proton excitations $\tilde{\nu}$. Particularly lucid results are obtained at $\tilde{x} \ll 1$. From (5), (6), and (7) we obtain

$$\tilde{\nu} = \left(1 - \frac{nM^2}{\mu^2}\right)\nu, \quad (6')$$

$$\mu = \left(1 + \sqrt{1 - \frac{4M^2 n}{\omega_o^2}}\right)\frac{\omega_o}{2}, \quad (7')$$

$$E - E_o = (\omega(k) - \epsilon_F)\nu. \quad (5')$$

It follows from (6') that $1 - nM^2/\mu^2 > 0$, and this, as can be readily seen, yields $\omega > \omega_0/2$ and corresponds to the sign chosen by us for the square root in (7'). If $\alpha_k = 4f^2k^2n/\omega_0^3 > 1$ for some interval $k_1 < k < k_2$, then a real solution of (7') exists only outside this interval. At the points $\alpha_k = 1$ we have $d\omega/dk = \infty$ and $\omega = \omega_0/2$. Thus, the condition $\alpha_k = 1$ corresponds to a discontinuity in the pion spectrum. Expression (5') has a clear physical meaning: the change of the system energy when one neutron is transformed into a proton and a pion excitation is equal to $\omega - \varepsilon_F$, as it should. Thus, in accord with [3], a second-order phase transition is possible only if $\omega(k) < \varepsilon_F$. It is indicated in [3] that in a realistic model this condition is not satisfied, at least up to very high nucleon densities.

We shall show that the correct expression for the pion frequency, corresponding to the Klein-Gordon-Fock equation, is

$$\omega^2 = \omega_o^2 - 2(n - 2\tilde{\nu})\, f^2k^2/\gamma\omega\,. \quad (8)$$

The average-field method is therefore unable to yield quantitative results.

3. Effective Lagrangian of Pion Field

To obtain the correct solution in the same physical model (nucleons + pion field, with wave vector $\vec{k}$), we can employ the following method: The Lagrangian of the problem is given by

$$L = \Sigma(w_n - E^{(n)})\Psi_n^+\Psi_n + \Sigma(w_p - E^{(p)})\Psi_p^+\Psi_p + (\omega^2 - \omega_o^2)\phi^+\phi + iMk(\Psi_n\vec{\sigma}\Psi_p^+\phi^+ - \Psi_n^+\vec{\sigma}\Psi_p\phi)\,, \quad (9)$$

where w_n, w_p, and ω are the field frequencies. It is easily seen that variation of L with respect to Ψ_n, Ψ_p, and ϕ yields the correct equation of motion (the Schrodinger equation for nucleons and the Klein-Gordon-Fock equation for pions). It is necessary to determine first the new energy eigenvalues of the neutrons and protons in a field ϕ with frequency ω. In analogy with (3) we obtain

$$\tilde{E}^{(n)} = E^{(n)} + \frac{\omega}{2}(1 - \gamma)\,, \quad \gamma = \sqrt{1 + \frac{8f^2k^2}{\omega^2}\phi^+\phi}\,, \quad (10)$$
$$\tilde{E}^{(p)} = E^{(p)} + \frac{\omega}{2}(1 + \gamma).$$

Substituting in (9), we get

$$L = \Sigma(w_n - E^{(n)})\tilde{n}^+\tilde{n} + \Sigma(w_p - E^{(p)})\tilde{p}^+\tilde{p} + (\omega^2 - \omega_o^2)\phi^+\phi + (n - 2\tilde{\nu}_p) \times \frac{\omega}{2}(\gamma - 1). \quad (11)$$

Variation of (11) with respect to $\phi^+\phi$ yields expression (8).

To obtain the energy from a Lagrangian that depends on the field frequencies (in our case, on w_n, w_p, and ω), we use the following rule [4]:

$$E = w_n\frac{\partial L}{\partial w_n} + w_p\frac{\partial L}{\partial w_p} + \omega\frac{\partial L}{\partial \omega} - L\,. \quad (12)$$

From (11), (12), and (8) we can easily obtain

$$E = E_o[(1 - \tilde{x})^{5/3} + \tilde{x}^{5/3}] + (3\omega^2 - \omega_o^2)\phi^+\phi\,. \quad (13)$$

The density of the "new" mesons (which equals the density of the "new" protons) is obtained from the expression

$$\tilde{\nu} = \frac{\partial L}{\partial \omega} = 2\omega\left[1 + \frac{\omega^2 - \omega_o^2}{\omega^2(\gamma+1)}\right]\phi^+\phi \ . \qquad (14)$$

At low density $\tilde{\nu}$, these equations again lead to an expression in the form (5')

$$E = (\omega(k) - \epsilon_F)\,\nu .$$

However, $\omega(k)$ should be determined from (8). This formula yields the frequency of the π^- mesons, and if ω is replaced by $-\omega$ it yields the π^+-meson frequency [2]. It is easy to see that as $\tilde{\nu} \to 0$ and if the condition

$$\xi = \frac{3\sqrt{3}f^2k^2n}{\omega_o^3} = 1$$

is satisfied, the sum of the π^-- and π^+-meson frequencies vanishes, i.e., the already mentioned instability for pair production sets in. Since pair production is not taken into account in our model, we can use it only for $\xi < 1$.

An analysis of equations (12), (13) and (14) at arbitrary density $\tilde{\nu}$ shows that no first- or second-order phase transitions occur in the considered model, up to very high density of nuclear matter, i.e.,

$$E(\tilde{\nu}) > E(0) .$$

It was suggested in [3] that proton stars ($\tilde{\nu}_p = \tilde{\nu}_\pi = n$) can exist. This suggestion is not corroborated in our model. It is difficult to say whether the result remains the same also in a more realistic calculation.

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GAUGE MODELS OF WEAK INTERACTION AND SUPERCHARGED HADRONS

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Submitted 23 July 1973
ZhETF Pis. Red. 18, No. 7, 447 — 451 (5 October 1973)

The possibility of searches for supercharged (charmed) hadron in a neutrino experiment and in PP interactions at high energies is discussed.

One of the most characteristic features of gauge models of weak interaction is the need **for** including supercharged (charmed) hadrons in the model (see the review [1] and the literature cited there). It follows from the absence of the $K_L \to 2\mu$ decay [1, 2] that supercharged hadrons R cannot be heavier than 7 - 10 GeV, and the mass difference between the K_L and K_S mesons yields even more stringent albeit less reliable estimates, namely $m_R \lesssim 2 - 3$ GeV (see also the review [3]).

The purpose of the present article is to emphasize that the need for using [2, 4] the Glashow-Iliopoulos-Maiani procedure [5] to eliminate neutral currents with $\Delta S = 1$ fixes rigidly the decay properties of the R-hadrons of lightest mass (heavy R-hadrons would decay strongly into lighter ones). Namely, the probabilities of of the β decays $R \to NI\nu$, $R \to YI\nu$, etc. should be comparable with the probabilities of the nonleptonic decays: