THE GELL-MANN — LOW FUNCTION IN SINGLE-CHARGE SCALAR QUANTUM FIELD THEORY

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The first three terms of the Gell-Mann — Low function are obtained in the $\lambda\phi^4$ theory. It is shown that the "zero-charge" situation remains in force when terms up to $\lambda^2[\lambda \ln(\lambda/k)]^4$ are taken into account.

An examination of the usual renormalized quantum field theories has shown that a general property of these theories is the "zero charge" situation, characterized by the appearance of a logarithmic pole in the scattering amplitude a(k) at momenta k >> m (m is the particle mass, e.g., in scalar field theory with interaction $H_{int} = \lambda\phi^4$

$$a(k^2) = \frac{a(m^2)}{1 - \frac{3}{2} a(m^2) \ln \frac{k^2}{m^2}} \quad . \tag{1}$$

The last formula was obtained in the principal logarithmic approximation and is valid when a(k) << 1. The renormalizability of the theory [1] allows us to introduce the Gell-Mann — Low function $\Psi(a)$, which describes the asymptotic behavior of the invariant charge at short distances.

$$\frac{da}{dx} = \Psi(a), \tag{2}$$

where $x = \ln(\lambda^2/k^2)$, and λ is the cutoff momentum. This function can be obtained with the aid of perturbation theory in the form of a power-series in a. The first term of this series

$$\Psi(a) = -\frac{3}{2}\frac{a^2}{16\pi^2} + \dots \tag{3}$$

leads to (1). The problem is to determine the change introduced into this solution by allowance for the last two terms of the expansion in $\Psi(a)$. In particular, if the Gell-Mann — Low function has a zero at the point a, then it can be readily seen from (2) that

$$a(k) \to a \quad . \tag{4}$$

This possibility was discussed in [2, 3]. The coefficient of the term a^3 in the function $\Psi(a)$ was determined, viz.,

$$\Psi(a) = -\frac{3}{2(16\pi^2)} a^2 + \frac{17}{6(16\pi^2)^2} a^3 - \dots \tag{5}$$

The question of the behavior of the Gell-Mann — Low function is of interest also in connection with the recent studies of Yang-Mills fields [4, 5]. It turns out that the ultraviolet behavior of the Green's functions in this theory is such that the ineraction of the vector mesons at short distances becomes asymptotically small, corresponding to the Bjorken scaling. In this theory, however, the vector mesons have no mass. One of the ways of overcoming this difficulty is to include an interaction between the vectors and scalar mesons; this interaction leads, via the Higgs mechanism, to a nonzero mass of the vector mesons.

As follows from (5), inclusion of a^3 leads to a zero of the function $\Psi(a)$. The next higher approximation, however, makes this zero disappear. We present here the result of a calculation of the Gell-Mann — Low function for the theory of n-component scalar field, accurate to terms of order a^4:

$$\Psi(a) = -\frac{(n+8)}{6(16\pi^2)} a^2 + \frac{3n+14}{6(16\pi^2)^2} a^3 - \frac{Da^4}{(16\pi^2)^3} + \dots \tag{6}$$

$$D = -\frac{19}{108}n^2 - \frac{191}{27}n - \frac{298}{27} - \frac{2}{9}(5n + 22)\zeta(3) - \frac{12n^2 + 55n + 186}{4 \cdot 81}\phi_3 +$$

$$+ \frac{(n+8)^2}{24 \cdot 81}\phi_1^2 + \frac{28n^2 + 53n - 108}{4 \cdot 81}\phi_1 - \frac{n^3 + 6n^2 + 150n + 572}{8 \cdot 81}\phi_2;$$

$$\phi_1 = -\sum_{i=1}^{3} \ln\frac{a_i^2}{q^2}; \quad \phi_2 = \sum_{i=1}^{3} \ln^2\frac{q_i^2}{q^2};$$

$\phi_3 = \sum_{i,j} f(p_i, q_j)$, with q_j the transferred momentum, q the renormalization momentum

$$f(p_i, q_j) = \int_0^1 \frac{dx}{x}\left[\frac{a^2 x(1-x)\ln(a^2 x(1-x))}{1 + a^2 x^2 - 2abx} - \right.$$

$$\left. - \frac{(1 + a^2 x - 2abx)\ln(1 + a^2 x - 2abx)}{1 + a^2 x^2 - 2abx}\right]$$

$a = p_i/q_j$, $b = \cos p_i\hat{\ }q_j$, and $\zeta(3)$ is a Riemann function.

As follows from (6), the coefficient of a^4 depends on the ratios of the external momenta. It is convenient to introduce a symmetrical normalization point, namely $p_i^2 \equiv p^2$, $q_i^2 \equiv (4/3)p^2$, and $b_i \equiv 1/\sqrt{3}$. At this point, the corresponding coefficient is equal to

$$D = -\frac{19}{108}n^2 - \frac{191}{27}n - \frac{298}{27} - \frac{2}{9}(5n + 22)\zeta(3) - \frac{3(12n^2 + 55n + 186)}{2 \cdot 81}I, \tag{7}$$

with $I = -0.740 \mp 0.001$. We thus get at $n = 1$

$$\Psi(a) = -\frac{3}{2}\frac{a^2}{16\pi^2} + \frac{17}{6}\frac{a^3}{(16\pi^2)^2} - 22\frac{a^4}{(16\pi^2)^3}. \tag{8}$$

It is easily seen from (8) that $\Psi(a)$ has no zero in this approximation. Obviously, this result cannot be reliable, and to ascertain the possible existence of a non-zero-charge situation it is desirable to investigate this question without the use of perturbation theory [6].

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EXPERIMENTAL OBSERVATION OF EMISSION LINES FROM AUTO-IONIZATION LEVELS OF CESIUM

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Information on the atomic auto-ionization states due to excitation of inner electrons have heretofore been extracted mainly from photoabsorption experiments. In a number of