$$\frac{1}{2\pi i}\left[T\left(\nu\sqrt{\frac{q^2+i\epsilon}{q^2}},\ q^2+i\epsilon\right)-T\left(\nu\sqrt{\frac{q^2-i\epsilon}{q^2}},\ q^2-i\epsilon\right)\right]_{\epsilon\to 0} =$$

$$= \int_0^1 d\alpha \int_0^\infty da\, h(\alpha,\ a)[\epsilon(q^2+\alpha\nu)\,\delta(q^2+2\alpha\nu-a)+\epsilon(q^2-\alpha\nu)\,\delta(q^2-2\alpha\nu-a). \tag{8}$$

On the other hand, we get from the DGSR the following expression for $w(\nu,\ q^2)$

$$w(\nu, q^2) = \int_0^1 d\alpha \int_0^\infty da\, h(\alpha,\ a)[\epsilon(q_0-p_0\alpha)\delta(q^2+2\alpha\nu-a)+\epsilon(q_0-p_0\alpha)\,\delta\times$$

$$\times(q^2-2\alpha\nu-a)].$$

It is easy to see that at $q^2 > 0$ and $\nu > 0$ formulas (8) and (9) are equal to each other, as well as to (1).

All the foregoing results can be directly applied to the scattering of a virtual photon by a nucleon, where at $q^2 < 0$ one measures the functions $w_1(\nu,\ q^2)$ and $w_2(\nu,\ q^2)$ in the process $e + N \to e +$ hadrons. The relations (2) - (5) hold in this case for the amplitudes $T_1(\nu,\ q^2)$ and $T_2(\nu,\ q^2) = [(\nu^2 - q^2)/q^2]T_1(\nu,\ q^2)$. In accord with the statements made above, it follows that from the known functions $w_1(\nu,\ q^2)$ and $w_2(\nu,\ q^2)$ at $q^2 < 0$ the amplitude $T_2(\nu,\ q^2)$ can be reconstructed uniquely at all real q^2 (since $T_2(m\sqrt{q^2},\ q^2) = 0$, which corresponds to the absence of subtractions in the dispersion relation in ν for $T_2(\nu,\ q^2)$, and the amplitude $T_1(\nu,\ q^2)$ accurate to a function of q^2, is analytic in the complex q^2 plane with a cut from 0 to ∞.[2])

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[1])We consider the dispersion relation in ν at $q'^2 < 0$. As is well known [2], the proof of the dispersion relation when the square of the mass is negative entails no difficulty).

[2])In practice, in view of the instability in the reconstruction of an analytic function from its values on the curve outside the cuts, one can obtain with sufficient accuracy only certain integrals of $T(\nu,\ q^2)$ with respect to q^2, with weighting functions of q^2.

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WAVE FUNCTIONS OF THE PARTON MODEL

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It is shown that the scaling of the cross sections of deeply inelastic processes means that the partons are localized in a three-dimensional Euclidean r-space that is the Fourier transform of the rapidities. The onset of scaling is determined by the form of the wave function of the hadron in this space.

1. In the momentum representation, the Schrödinger wave function of the hadron

$$\phi_n(k_1\epsilon_1;\ \ldots\ k_n\epsilon_n;\ M)$$

depends on the momenta k_i and the energies ϵ_i of the partons, with

$$\epsilon_i \equiv \sqrt{k_i^2 + m_i^2}$$

where m_i is the parton mass. The hadron momentum is equal to the sum of the parton momenta, and its mass satisfies the inequality

$$M < \sum_{i=1}^{n} m_i ,$$

which is equivalent to stating that the free hadron cannot decay into partons. Since we are dealing with the bound state, the wave functions ϕ_n should be regarded as square-integrable. The relativistically-invariant norm is determined by the integral

$$\int |\phi_n|^2 \prod_{i=1}^{n} \frac{dk_i}{\epsilon_i} < \infty .$$

The physical nature of the partons will no longer be of importance to us. If we so desire, we can take the partons to be quarks. We can interpret ϕ_n simply as the components of a Fock vector of state of a system of quantum fields. In this case the role of the parton can be assumed by any sufficiently heavy and long-lived particle. In the present article, the spin of the parton can be regarded as equal to zero. This limitation is not fundamental and is used only to simplify the formulas.

Under Lorentz transformations the functions ϕ_n are transformed in accord with a certain representation T_g as a result of the shift of the arguments. This representation is reducible and unitary, since the relativistically invariant norm of the functions ϕ_n is positive-definite. The unitary and irreducible representations contained in T_g (we recall that all these representations are infinitely-dimensional, owing to the non-compactness of the Lorentz group). According to [1, 2], the decomposition of the Hilbert space of the functions ϕ_n into parts that are irreducible with respect to the Lorentz group is effected by the following integral transformations (we use the notation introduced in [3]):

$$\phi(k, \epsilon) = (2\pi)^{-3/2} \int \xi(k, r) \psi(r)\, dr , \tag{1}$$

$$\psi(r) = m(2\pi)^{-3/2} \int \xi^*(k, r)\, \phi(k, \epsilon) \frac{dk}{\epsilon} . \tag{2}$$

Here

$$\xi(k, r) = \left(\frac{\epsilon - kn}{m}\right)^{-1+mr}, \quad n \equiv r/r . \tag{3}$$

The basis functions ξ are orthogonal and normalized to delta functions, so that

$$\int |\psi|^2 dr = \int |\phi|^2 \frac{dk}{\epsilon} .$$

We have presented, for brevity, formulas for functions ϕ of one vector argument. The corresponding expressions for the functions ϕ_n are obtained by n-fold application of formulas (1) and (2). The function $\psi(r)$ is transformed in accordance with the irreducible unitary representation of the Lorentz group. Then the angle argument n experiences a shift, the modulus of r remains unchanged, and the entire function acquires a factor

$$\psi'(r') = \xi^*(P, r')\, \psi(u_\beta^{-1} r'), \quad r' = r . \tag{4}$$

Here P is the momentum of a parton with velocity β (we assume that the new system moves with velocity $-\beta$), and u_β is the rotation matrix connecting the vectors n' and n:

$$n' = u_\beta n .$$

This rotation is equal to the aberration of the light ray: n' is parallel to the direction of a light wave propagating in the initial system along n. It follows from the transformation law (4) that if the angle part of the function $\psi(r)$ is separated in the rest system of the hadron, then the entire inforamtion on the dynamics of the parton interaction, information concentrated in the radial functions, is transformed into any Lorentz reference frame in almost the same manner as in rotations, since the radial variables r are Lorentz-invariant, and the factors ξ^* do not depend on the form of the ψ function. The situation is different in the case of the wave functions ϕ in the momentum representation, namely, the separation of the angular part in some reference system changes little (in the sense of the definiteness of the transformation properties), since the Lorentz transformation changes the absolute values of the momenta. It can be stated that the transition to r-space results in a maximum separation of the kinematic dynamic characteristics of the system of relativistic particles that are described by the wave function.

2. We indicate one more important property of the r-representation, first noted in [3], namely, in the nonrelativistic limit ($k/m \ll 1$) formulas (1) and (2) go over into Fourier integrals, and therefore r is transformed into the usual radius vector of the point. In the general case, on the other hand, the r-space is the Fourier transform of the rapidities. This is easiest to see if it is assumed that in the hadron rest system all the partons have zero orbital angular momenta. Then the ψ function in this system depends only on the radial variables, and in formula (1) it is easy to carry out the integration with respect to the angles. It yields

$$\phi(k, \epsilon) = \frac{1}{\operatorname{sh} \eta_k} \sqrt{\frac{2}{\pi}} \int_0^\infty \sin \eta_k r \, \chi(r) \, dr \, . \qquad (5)$$

We have put here $m = 1$ and $\chi(r) = r\psi(r)$, and have denoted by η_k the rapidity of a parton with momentum k:

$$\eta_k = \frac{1}{2} \ln \frac{\epsilon + k}{\epsilon - k} \, .$$

Using the transformation law (4) and the addition theorem for the basis functions ξ (see [2]), we can show that in a reference frame in which the hadron moves with velocity β we obtain an analogous relation that differs from (5) in that η_k is replaced by η, where η is given by

$$\operatorname{ch} \eta = \epsilon (P^2 + 1)^{1/2} - kP. \qquad (6)$$

We put

$$k = xP + k_t \, , \qquad k_t P = 0$$

and assume that

$$0 < x < 1, \qquad xP \gg 1, \quad k_t \, . \qquad (7)$$

Then

$$\eta \approx \eta_0 + 0(P^{-2}) \approx \eta_p - \eta_k \, , \qquad (8)$$

where

$$\eta_0 = \operatorname{arc\,ch} \frac{x^2 + k_t^2 + 1}{2x} \qquad (9)$$

and η_p is the rapidity of a hadron with velocity β. From (8) and (9) it follows that as $P \to \infty$ the quantities η cease to depend on P. At large but finite P we can leave out the remainder $0(P^{-2})$ from under the integral sign in (5) if the effective integration interval is finite and small enough. If this is the case, then the distribution of the partons with respect to x and k_t does not depend on P. It is well known (see, e.g., [4, 5]) that this gives rise to scale invariance of the cross section of deeply inelastic interaction of leptons with hadrons. The quantitative criteria for the "onset of scaling" should contain, besides the kinematic conditions (7), also the dimensions of the partons in r-space (more accurately, the moments $\langle r^n \rangle$).

3. We have considered above spinless partons in s-states[1)]. The foregoing remains valid also for partons with spin and with nonzero orbital angular momenta (we indicate that formulas for particles with spin, analogous to (1) and (2), have been obtained in [6, 7]; a useful formalism for calculations in the r-representation has been developed in [3, 8 - 10]).

We do not discuss in this article the spectrum and multiplicity of the pions produced in deeply-inelastic scattering. We note that the pion rapidity distribution should also be characterized by scaling properties analogous to those that follow from (5) - (9). This is due to the finite dimension of the parton in r-space (just as the scaling distribution of the partons is a consequence of the limited dimension of the target hadron). The multiplicity is a more complicated problem, which cannot be solved without involving additional hypotheses. Since r-space is Fourier-conjugate to the rapidities, it is clear that the simplest models for the ψ functions will lead to a logarithmic connection between the multiplicity and the energy transfer. In particular, within the framework of such an approach, it would be quite natural for the multiplicity to increase like some power (preferably the cube) of the logarithm of the energy transferred in the deeply inelastic process. It is planned to consider these questions, as well as the already mentioned extension of formula (5) to the case of partons with nonzero spin, in other papers.

We emphasize in conclusion that the r-space is similar in many of its properties to the space of ordinary coordinates, and this may be useful both from the heuristic point of view and as a means of simplifying the formalism. In particular, different variants of the "onset of scaling" or of the behavior of the cross section in the boundary region of small values of x ("wee"partons) may reduce, within the framework of the r-representation, to a relatively simple parametrization of the ψ function (an asymptotic form or behavior at zero). New possibilities are uncovered by the use of the r-representation also in the theory of collisions of nuclei moving with relativistic velocities.

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1)After the completion of this work, I learned that a similar Fourier transformation in the rapidity was considered independently by Ya. A. Smorodinakii.

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EFFECTS OF BRANCH CUTS ON THE CORRELATIONS BETWEEN CHARGED AND NEUTRAL PIONS

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It is shown that the experimentally observed correlation between the number of charged and neutral pions, and also the scaling of Koba, Nielsen, and Olesen, can be attributed to the contribution of branch points.

Experiments performed at Serpukhov, Batavia, and CERN have shown that the average number $\langle n^0(n^-)\rangle$ of π^0 mesons depends on the number n of the pions produced in the given event [1]. With increasing energy, the correlation increases and at $\sqrt{s}$ = 53 GeV we have $\langle n^0(n^-)\rangle \simeq 0.8n^-$. This behavior of $\langle n^0(n^-)\rangle$ can be attributed to contributions of branch cuts, i.e., processes in which not one but several multiperipheral "ladders" are emitted (Fig. 1). This is easiest to