deeply-inelastic scattering. We note that the pion rapidity distribution should also be characterized by scaling properties analogous to those that follow from (5) - (9). This is due to the finite dimension of the parton in r-space (just as the scaling distribution of the partons is a consequence of the limited dimension of the target hadron). The multiplicity is a more complicated problem, which cannot be solved without involving additional hypotheses. Since r-space is Fourier-conjugate to the rapidities, it is clear that the simplest models for the $\psi$ functions will lead to a logarithmic connection between the multiplicity and the energy transfer. In particular, within the framework of such an approach, it would be quite natural for the multiplicity to increase like some power (preferably the cube) of the logarithm of the energy transferred in the deeply inelastic process. It is planned to consider these questions, as well as the already mentioned extension of formula (5) to the case of partons with nonzero spin, in other papers.

We emphasize in conclusion that the r-space is similar in many of its properties to the space of ordinary coordinates, and this may be useful both from the heuristic point of view and as a means of simplifying the formalism. In particular, different variants of the "onset of scaling" or of the behavior of the cross section in the boundary region of small values of x ("wee"partons) may reduce, within the framework of the r-representation, to a relatively simple parametrization of the $\psi$ function (an asymptotic form or behavior at zero). New possibilities are uncovered by the use of the r-representation also in the theory of collisions of nuclei moving with relativistic velocities.

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[1]After the completion of this work, I learned that a similar Fourier transformation in the rapidity was considered independently by Ya. A. Smorodinakii.

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## EFFECTS OF BRANCH CUTS ON THE CORRELATIONS BETWEEN CHARGED AND NEUTRAL PIONS

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It is shown that the experimentally observed correlation between the number of charged and neutral pions, and also the scaling of Koba, Nielsen, and Olesen, can be attributed to the contribution of branch points.

Experiments performed at Serpukhov, Batavia, and CERN have shown that the average number $\langle n^0(n^-)\rangle$ of $\pi^0$ mesons depends on the number n of the pions produced in the given event [1]. With increasing energy, the correlation increases and at $\sqrt{s}$ = 53 GeV we have $\langle n^0(n^-)\rangle \simeq 0.8n^-$. This behavior of $\langle n^0(n^-)\rangle$ can be attributed to contributions of branch cuts, i.e., processes in which not one but several multiperipheral "ladders" are emitted (Fig. 1). This is easiest to illustrate with Fig. 1a as an example.

Assume that the meson emission in each short ladder is independent (in the sense of Poisson) and that the average number of pions generated in one ladder $N = \langle n_1^- \rangle = \langle n_1^+ \rangle = \langle n_1^0 \rangle$ is large. Then peaks will be observed in the cross section $\sigma_{n^-}$ for the production of $n^-$ negative pions [2], at $n^- = qN$ ($q = 1, 2, 3, \ldots$); these peaks correspond to processes with emission of one, two, three... ladders (Fig. 2a). Since the width of the Poisson distribution is proportional to $\sqrt{\langle n \rangle}$, and the distance between the maxima is N, the peaks do not overlap in practice (at ultrahigh energies, when N is large).

On the other hand, the average number of $\pi^0$ produced by the q-ladders is equal to qN, therefore the function $\langle n^0(n^-)\rangle$ shown in Fig. 2 will have steps in the regions $n^- \simeq qN$, $\langle n^0 \rangle \simeq qN \simeq n^-$.

In the intervals between the maxima of $\sigma_{n^-}$, the main contribution will be made by the enhanced graphs [2, 3] shown in Figs. 1b and 1c (dashed in Figs. 2a and 2c). If the three-pomeron vertex $g_{PPP} \to 0$ as $t \to 0$, then the probability of such processes is smaller by a factor $\ln^m(s)$ (where $m \geq 1$ and s is the square of the total energy in the c.m.s.) than the contribution of the non-enhanced branch points (Fig. 1a), but on the other hand the average number of generated pions can now be arbitrarily, and not merely a multiple of N.

The average number of pions emitted in the diagrams 1b and 1c is proportional to the total "length" of the reggeons and depends on the positions of the vertices $g_1$, $g_2$, ..., in rapidity space. And since integration is carried out over the rapidities $\eta_i$ of the vertices $g_i$, one can always find a process for which $n^- = \langle n \rangle$, and consequently also $\langle n^0 \rangle = \langle n \rangle = n^-$, and it is just this process which will make the main contribution to $\sigma_{n^-}$ (provided only that $n^-$ does not fall in the intervals $qN \pm \sqrt{2qN}$). As a result we obtain the curve of Fig. 2b.

It may turn out (for example, if the vertex $g_{PPP}$ does not vanish as $t \to 0$) that the enhanced graphs are important for arbitrary $n^-$, including $n^- \simeq qN$. In this case the steps on Fig. 2b become smoothed out and the function $\langle n^0(n^-)\rangle$ becomes simply the straight line $\langle n^0(n^-)\rangle = n^-$ (Figs. 2c and 2d). The same occurs at contemporary energies, when N is not large and the peaks of the non-enhanced branch points overlap strongly.

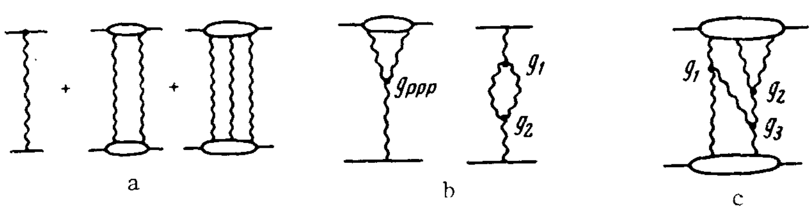


Fig. 1

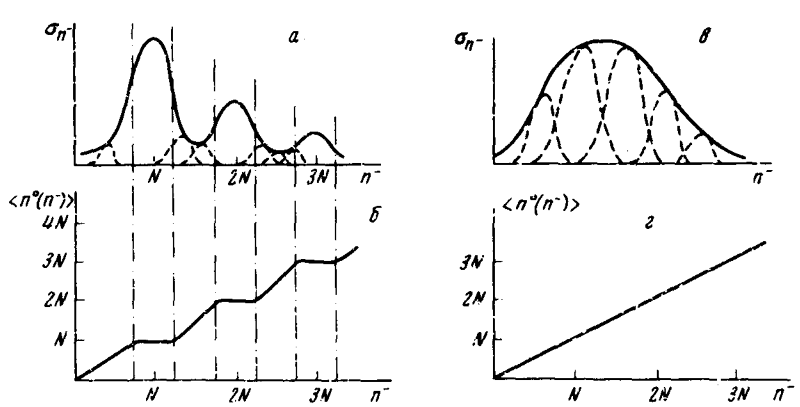


Fig. 2

We shall now prove that the enhanced graphs indeed lead to the 100% correlation $\langle n^0(n^-)\rangle = n^-$. Let the probability of a process in which an average of $\langle n \rangle = yN$ negative pions are emitted be given by the function $f(y)$ at a fixed initial energy s, and let the multiplicity in each process have a Poisson distribution.

Then the topological cross section is

$$\sigma_{n^-} = \sigma_{tot} \int f(y)\, e^{-yN} (yN)^{n^-} \frac{dy}{(n^-)!}, \tag{1}$$

and the average number of neutral pions is

$$\langle n^0(n^-)\rangle = \frac{\sigma_{tot}}{\sigma_{n^-}} \int yN f(y)\, e^{-yN} (yN)^{n^-} \frac{dy}{(n^-)!}. \tag{2}$$

If the function $f(y)$ varies sufficiently slowly, then the integrals (1) and (2) can be determined by the saddle-point method, by replacing $(yN)^{n^-}\exp(-yN)$ by

$(n^-/e)^{n^-}\exp(-(y - y_0)^2N^2/2n^-)$, and assuming that $f(y)$ and $yf(y)$ remain practically unchanged in the essential region of integration

$$|y - y_0| < \sqrt{2n^-}/N; \quad (y_0 = n^-/N), \text{i.e.}, f'(y)/f(y) << N/\sqrt{2n^-}. \tag{3}$$

Integrating, we obtain

$$\sigma_{n^-} = \frac{\sigma_{tot}}{N} f\left(y_0 = \frac{n^-}{N}\right) \tag{4}$$

and

$$< n^0(n^-) > = y_0 N = n^- . \tag{5}$$

The condition (3), under which formulas (4) and (5) are valid, means that the variation of $f(y)$ within the limits of the width of the Poisson peak ($\Delta\langle n\rangle = \sqrt{2\langle n\rangle}$) can be neglected. For enhanced graphs this condition is satisfied with a large margin, since the natural scale of the derivative [3] $f'(y)/f(y) \simeq 1 << N/\sqrt{2n^-}$ ($N \propto \ln s$). We have thus shown that allowance for the branch cuts leads to a single correlation $\langle n^0\rangle = n^-$ [Eq. (5)]. Since the isotopic invariance leads to $\langle n^0\rangle = n^-$ in each graph, this conclusion is valid also when each ladder has its own charge correlations between the neutral and negative pions (provided the $\sigma_{n^-}$ distribution for each individual graph is narrow enough (with width on the order $\sqrt{\langle n\rangle}$), so as to be able to use the saddle-point method). Let us return to expression (4). Here $f(y)$ is, in the general case, a function of two arguments, of $y$ and of the initial energy $s$ (more accurately $(\alpha' \ln s + R^2)$ [2, 3], where $R^2$ is the radius in the form factors of the vertices $g_i$). But since $\alpha'$ of a vacuum pole is small, we can neglect at present-day energies the dependence of $f(y, \ln s)$ on $\ln s$ ($\alpha' \ln s < R^2$). At the same time, the dependence of $f(y)$ on $y$ remains in force, since at different values of $y = n^-/N$ different graphs (Fig. 1) contribute to $f(y)$. For example, the diagrams 1b describe processes in which from 0 to 2N negative pions are produced, and diagram 1c describes processes with emission from 0 to 4n negative pions. Thus, in the region of attainable energies formula (4) explains [4] the experimentally observed scaling of Koba, Nielsen, and Olesen [5], according to which the ratio $N\sigma_{n^-}/\sigma_{tot} = f(n^-/N)$ does not depend on the energy $s$. In addition, Egs. (1) and (2) lead directly to a relation between $\sigma_{n^-}$ and $\langle n^0\rangle$:

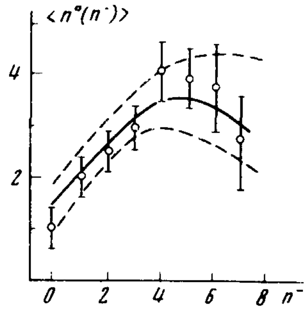


Fig. 3. Average number of $\pi^0$ mesons vs. the number of $\pi^-$ mesons in pp collisions at 205 GeV/c [6]. Solid curve — calculation with formula (6), using the experimental values of $\sigma_{n^-}$ [7]. The dashed lines show the error corridor.

$$< n^0(n^-) > = (n^- + 1)\sigma_{n^- + 1}/\sigma_{n^-} . \tag{6}$$

Although relation (6), strictly speaking, is valid only if the particle-number distribution in each ladder obeys the Poisson law, it is seen from Fig. 3 that the agreement with experiment is fair.


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