CRITICAL POINT OF PHASE TRANSITIONS OF FIRST ORDER BUT CLOSE TO SECOND ORDER

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It is known that in certain ferroelectrics and magnetics the phase transition is of first order but is close to second order. Larkin and Pikin [1] have shown for the elastic-isotropic model that the first-order transition is connected with the compressibility of the lattice and with the influence of the long-wave phonons in the case when the specific heat becomes infinite in the absence of this influence.

In real experiments there are always factors that "smear out" the peak of the specific heat, namely the electric field E or the magnetic field H of the impurity. We shall show that there exist critical values of the parameters, such that the phase transitions of the described type vanish; we shall obtain the critical field H_c and the critical concentration γ_c, and investigate the thermodynamics near such a critical point.

To simplify the exposition, we confine ourselves to magnetic materials. We use as the models the Ising model in a magnetic field (MIM) and the decorated Ising model (DIM) [2]. In the former case, the Hamiltonian, with allowance for the compressibility, takes the form

$$\mathcal{H} = \sum_{ij}\left[\left(\frac{K_0}{2}-\frac{\mu}{3}\right)\left(\frac{\partial u_\alpha}{\partial r_\alpha}\right)^2 + \mu\left(\frac{\partial u_\alpha}{\partial r_\beta}\right)^2 + I\left(1+q\frac{\partial u_\alpha}{\partial r_\alpha}\right)\sigma_i\sigma_j\right] + \sum_i Hm\sigma_i , \tag{1}$$

and in the latter case

$$\mathcal{H} = \sum_{ij}\left[\left(\frac{K_0}{2}-\frac{\mu}{3}\right)\left(\frac{\partial u_\alpha}{\partial r_\alpha}\right)^2 + \mu\left(\frac{\partial u_\alpha}{\partial r_\beta}\right)^2 + I\left(1+q\frac{\partial u_\alpha}{\partial r_\alpha}\right)\sigma_i\pi\right]. \tag{2}$$

Here K_0 and μ are the moduli of hydrostatic compression and shear, $\partial u_\alpha/\partial r_\beta$ the strain tensor, I the exchange integral, $q = (\sigma/3I)(\partial I/\partial a)$, a the interatomic distance, $\sigma_i = \pm 1$, π; $\sigma_j = 0, \pm 1$, and m is the magnetic moment of the site.

We perform the transformation described in [1] and expand the expression for free energy (Eq. (3.1) of [3]) in integer powers of τ, obtaining the Gibbs potential Φ for the MIM in parametric form[1)]:

$$\Phi = \Phi_0 - \frac{p^2}{2K_0} + T_c\left[ah^\rho - \frac{bh^{-\epsilon}}{2}x^2 + \frac{dh^{-\phi}}{12}x^4 + \frac{\lambda}{2}\left(ah^\rho - bh^{-\epsilon}x + \frac{dh^{-\phi}}{3}x^3\right)^2\right], \tag{3}$$

$$t \equiv \frac{T - T_c - cp}{T_c} - \lambda a h^\rho = x - \lambda b h^{-\epsilon} x + \frac{\lambda d h^{-\phi}}{3}x^3, \tag{4}$$

where Φ is a smooth function of the temperature T and of the pressure p, H_0 =

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1)It should be noted that it is impossible to make a continuous transition to the limit $h \to 0$ in (3), since any field is strong in a region sufficiently close to the Curie point.

I/m, $h = H/H_0$ is the dimensionless magnetic field, $\lambda = 4\mu K_0 c^2/[T_c(3K_0 + 4\mu)]$, $c = (q/K_0)(\partial T_c/\partial a)$, z is a parameter, T_c is the transition temperature, a, b, $d \sim 1$, $\rho = \nu_2/(6 - \nu_1)$, $\varepsilon = (6 - 2\nu_2)/(6 - \nu_1)$, $\phi = (18 - 4\nu_2)/(6 - \nu_1)$, and the critical exponents ν_1 and ν_2 are defined by the relations $\langle\langle m(\vec{r})m(\vec{r}\,')\rangle\rangle \sim |\vec{r} - \vec{r}\,'|^{-\nu_1}$ and $\langle\langle \mathscr{E}(\vec{r})\mathscr{E}(\vec{r}\,')\rangle\rangle \sim |\vec{r} - \vec{r}\,'|^{-\nu_2}$. For the three-dimensional Ising model we have $\nu_1 \simeq 1$ and $\nu_2 \simeq 2.8$.

It follows from (4) that x, meaning also Φ, becomes a non-unique function of the temperature when $h < h_c = (\lambda b)^{1/\varepsilon}$, with $x_1 \to x_2$ as $h \to h_c$ (x_1 and x_2 are the roots of (4)).

Formulas (3) and (4) make it possible to construct the thermodynamics of the system near the critical point in the (h, t) plane. For simplicity let us consider the thermodynamic quantities along the lines t = 0 and $h = h_c$ ($\tilde{y}$ denotes the singular part of y, $\eta = (H_c - H)/H_c$, and $g_k \sim 1$).

For $t \to 0^{\pm}$, $\eta > 0$,

$$\tilde{S}_{\pm} = \pm g_1(\lambda b)^{(\phi/2\epsilon)-(3/2)}\eta^{1/2}, \quad \tilde{M}_{\pm} \equiv \left(-\frac{\tilde{\partial}\phi}{\partial H}\right)_{\pm} = \pm\frac{g_2 T_c}{H_o}(\lambda b)^{\nu_1/(6-\nu_1)}\eta^{1/2},$$

$$\tilde{c}_p = \frac{\eta^{-1}}{\lambda}, \qquad \tilde{\chi} \equiv \left(\frac{\tilde{\partial} M}{\partial H}\right) = \frac{g_3 H_o}{T_c}(\lambda b)^{-(6-2\nu_1)/(6-2\nu_2)}\eta^{-1}, \tag{5}$$

$$\left(\frac{\partial \tilde{c}_v}{\partial T}\right)_{\pm} = \pm\frac{g_4}{T_c}\left(\frac{4\mu}{3K_o + 4\mu}\right)^2(\lambda b)^{(-\phi/2\epsilon)-(1/2)}\eta^{-1/2}.$$

For $\eta = 0$

$$\tilde{c}_p = g_5(\lambda b)^{(\phi/3\epsilon)-(4/3)}t^{-2/3}, \quad \left(\frac{\partial \tilde{c}_v}{\partial T}\right) = \frac{g_6}{T_c}\left(\frac{4\mu}{3K_o + 4\mu}\right)^2(\lambda b)^{(-\phi/3\epsilon)-(2/3)}t^{-1/3}. \tag{6}$$

For the DIM we obtain for small γ, using (1) and (2),

$$\Phi = \Phi_o - \frac{p^2}{2K_o} + T_c\left[\frac{Ax^2}{2\gamma} + \frac{1-a}{2-a}\frac{B|x|^{(2-a)/(1-a)}}{\gamma^{(2-a)/(1-a)}} + \frac{\Lambda}{2}\left(\frac{Ax}{\gamma} - \frac{Bx|x|^{a/(1-a)}}{\gamma^{(2-a)/(1-a)}}\right)^2\right],$$

$$\tau \equiv \frac{T - T_c - cp}{T_c} = x - \frac{\lambda Ax}{\gamma} + \frac{\lambda Bx|x|^{a/(1-a)}}{\gamma^{(2-a)/(1-a)}}. \tag{7}$$

Here γ is the concentration of the nonmagnetic impurities, A, B $\sim$ 1, and α is the critical exponent of the specific heat in the Ising model, $\alpha = (6 - 2\nu_2)/(6 - \nu_2)$. The first-order phase transition vanishes at $\gamma > \gamma_c = \lambda A$. The singular parts of the thermodynamic quantities, obtained from (5) at t = 0 and $\xi = (\gamma_c - \gamma)/\gamma_c > 0$, are given by

$$\tilde{S}_{\pm} = \pm g_8(\lambda A)^{(1-a)/a}\xi^{(1-a)/a}, \quad \tilde{c}_p = \frac{\xi^{-1}}{\lambda},$$

$$\left(\frac{\partial \tilde{c}_k}{\partial T}\right)_{\pm} = \pm \frac{g_9}{T_c}\left(\frac{4\mu}{3K_0+4\mu}\right)^2 (\lambda A)^{-X-1}\xi^{(a-1)/a}, \quad (8)$$

and for $\xi = 0$, $r \neq 0$:

$$\tilde{c}_p = g_{10}(\lambda A)^{a/(1-a)}|r|^{-a}, \quad \left(\frac{\partial \tilde{c}_v}{\partial T}\right) = \frac{g_{11}}{T_c}\left(\frac{4\mu}{3K_0+4\mu}\right)^2 (\lambda A)^{1/(1-a)}\frac{|r|^a}{r}. \quad (9)$$

It is clear from formulas (3) and (9) that in our case the thermodynamics differs appreciably from the classical one near the critical point of a liquid-gas system (4). For the MIM, in particular, c_p is proportional to $t^{-2/3}$ and not to t^{-1}.

The region of applicability of formulas (6) and (8) is bounded by the conditions $|t| << (\lambda b)^{1/\alpha}$ and $|\tau| << (\lambda A)^{1/\alpha}$ for the MIM and DIM, respectively. Such a strong dependence on λ ($1/\alpha \simeq 8$ for the three-dimensional Ising model) makes it possible to observe experimentally the described effects only in substances with λb (or λA) sufficiently close to unity.

For a similar reason ($1/\varepsilon \simeq 12$), the first-order transitions described in [1] are suppressed even by weak magnetic fields.

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HALL EFFECT IN SEMICONDUCTORS IN A STRONG ELECTRIC FIELD

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When a semiconductor is placed in crossed electric and magnetic fields then a current or a potential difference is produced in the third direction, depending on the boundary conditions. This is the well-known Hall effect. It is also known that in strong electric fields the Hall coefficient begins to depend on the field and can strongly deviate from the value in weak fields. Moreover, it was observed that in p-tellurium [1] the sign of the Hall constant is even reversed (the type of conductivity remaining the same). On the other hand, the current-voltage characteristics of crystals placed in a strong magnetic field have revealed not ordinary current saturation but the inverse effect, voltage saturation [2, 3]. Moreover, Moore has observed [3] a gradual transition from current saturation to voltage saturation with changing magnetic field. We shall show below that all these effects are of the same nature and are closely linked to phonon generation by supersonic carrier motion in a strong electric field [4].