We note that the expression for E_{es} in [1] contains only the first two terms of (7). We shall now show that the sum of the last two terms in (7) is equal to zero. Indeed, the contribution made to the sum over the quasicontinuum by the reciprocal-lattice sites is negligibly small ($\sim 1/N$) and therefore we can replace this sum by the integral over all of reciprocal space. The latter can be readily evaluated, and as a result we find that the last two terms in (7) cancel each other exactly. Finally we get

$$E_{es} = \bar{E}_{es}. \tag{9}$$

We recall, however, that relation (9) was obtained under the assumption that there is no short-range order at all in the alloy. Allowance for the short-range order leads immediately to the appearance of an additional contribution to the electrostatic energy

$$\Delta E_{es} = \frac{1}{2}(Z_A - Z_B)^2 \left\{ \sum_{R} \epsilon(R) \frac{1 - \mathrm{erf}[\sqrt{\eta}|R|]}{|R|} + \frac{4\pi}{\Omega} \sum_{q}{}'' \frac{1}{q^2} e^{-q^2/4\eta} \right\}. \tag{10}$$

Here $\epsilon(R)$ are the correlation parameters defined by the relation [4]

$$\overline{C_R C_{R'}}_{R \neq R'} = c^2 + \epsilon(R - R'), \quad \epsilon(q) = \frac{1}{N} \sum_{R \neq 0} \epsilon(R) e^{iqR}.$$

It is easy to verify that ΔE_{es} does not depend on η ($\partial \Delta E_{es}/\partial \eta = 0$), as should be the case in accordance with the requirements of the Ewald transformation.

In conclusion we note that the results are valid for both a Bravais lattice and for a lattice with a basis.

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PLANAR MODEL OF A BRANCHED DOMAIN STRUCTURE OF A CUBIC FERROMAGNET

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It is well known that in cubic ferromagnets cut in the form of plates perpendicular to the easy axis, a domain structure with closed triangles is possible [1]. There is no anisotropy energy in this case, so that the total energy is made up of the energy of the surface tension on the phase boundaries and the magnetostriction energy, which is concentrated near the surface of the sample at distances on the order of the domain widths. The magnetostriction energy is very small in most cases (e.g., in iron). The total energy is therefore also small, and as a result this structure hardly ever becomes branched. According to an estimate by Lifshitz [1], branching in an iron plate cut perpendicular to the easy axis should begin only when the plate thickness is

$\ell > 10^4$ cm. In the case of an unbranched structure, the domain thickness is $a \sim \sqrt{\ell}$.

A simplest generalization of this structure to the case when the surface of the plate is inclined to the easy eaxis is the structure shown in Fig. 1. The anisotropy energy in the triangular domains is $U_{an} = (\beta/2)M^2\sin^2\gamma\cos^2\gamma$, where M is the magnetization. In iron we have $\beta = 0.28$. This structure is not branched, so that $a \sim \sqrt{\ell}$.

If $\gamma \sim 1$, the structure shown in Fig. 1 is not favored energywise. The total energy can be appreciably decreased if the branch structure shown in Fig. 2 is formed. This figure shows only two successive stages of branching. Actually, the breakup continues until the dimensions of the produced domains become comparable with the thickness of the 180° boundary δ_{180} (the latter is much larger than the thickness of the 90° boundary δ_{90}). This makes it possible to estimate the number of branchings:

$$n \sim \frac{\ln a/\delta_{180}}{\ln 3}.$$

We can thus eliminate the anisotropy energy almost completely. This energy is concentrated only in a narrow layer near the sample surface (at a distance on the order of δ_{180}).

The energy of such a structure per unit plate-surface area (taking both sides of the plate into account) is

$$E = \left[\frac{4}{3\ln 3}(2\sqrt{2}\Delta_{90} + \Delta_{180})\ln\frac{a}{\delta_{180}} + \frac{2}{3}kM^2a + \frac{\Lambda_{180}(\ell - 2a)}{a}\right]\cos\gamma. \qquad (1)$$

The term proportional to $\ln(a/\delta_{180})$ is the energy of the branched boundaries (Δ_{90} and Δ_{180} are the energies of the 90° and 180° boundaries and have been calculated in [1, 2]: $\Delta_{90} = 0.863\Delta_{180} \sim \beta\delta_{90}M^2$). After each branching, the dimensions of the domains are decreased by a factor of three, and their number increases accordingly, so that the total energy of the branched boundaries is proportional to the number of branchings n. The branching is favored energywise if this energy is smaller than the anisotropy energy of the triangular domains shown in Fig. 2. When $\gamma \sim 1$ and $a > \delta_{180}$, this condition is always satisfied.

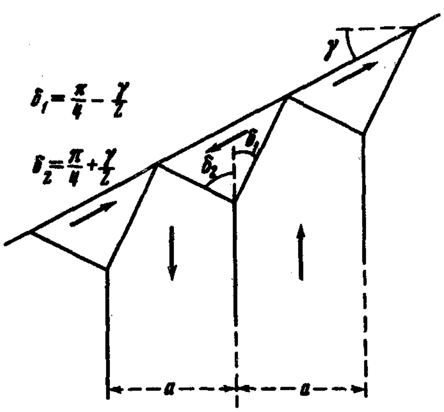

Fig. 1

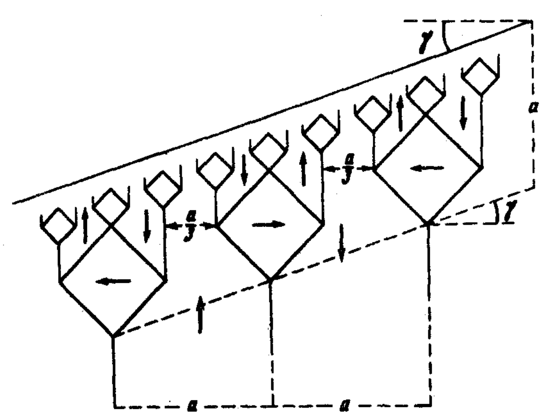

Fig. 2

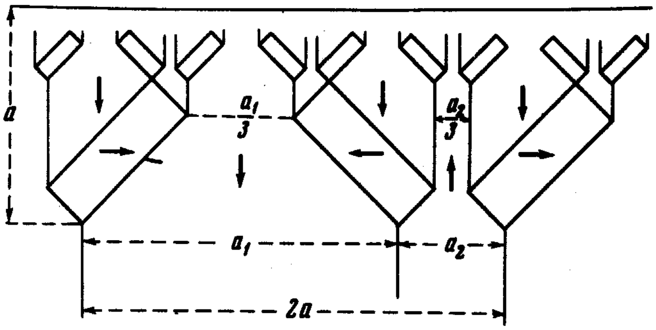

Fig. 3

The second term is the magnetostriction energy. We estimate it by assigning to the quadrangular domains the effective energy of uniaxial anisotropy $U_{m.e.} = kM^2$ [1]. In iron, $k = 3.3 \times 10^{-4}$. We thus obtain an upper limit for the magnetostriction energy of the entire body [1].

The last term is the energy of surface tension on the unbranched boundaries (the plate thickness ℓ is measured along the easy axis).

Minimizing this expression, we obtain a quadratic equation whose positive root is

$$a = 0{,}466\ell[1 + (1 + \ell/\ell_{k1})^{1/2}]^{-1}\,, \tag{2}$$

$$\ell_{k1} = \frac{2\sqrt{2}\,\Delta_{90} + \Delta_{180}}{kM^2 \ln 3} \sim \delta_{90}\frac{\beta}{k} \sim \delta_{180}\frac{\beta}{k \ln \beta/k}\;. \tag{3}$$

In the case of iron, $\ell_{cr1} = 3 \times 10^{-3}$ cm.

Thus, at small thicknesses $\ell \ll \ell_{cr1}$ a linear relation should be obtained, $a = 0.233\ell$, whereas for $\ell \gg \ell_{cr1}$ this dependence is of the square-root type, $a = 0.466\sqrt{\ell_{cr1}\ell}$, in spite of the fact that the structure is branched. In the latter case, the energy is $E = 0.62kM^2\sqrt{\ell_{cr1}\ell}$. At very large values of ℓ one should observe a maximally branched structure, with $a \sim \ell^{2/3}$, of the type calculated in [3]. The energy of such a structure is of the order of $\beta\delta_{90}^{2/3}\ell^{1/3}M^2$. Comparing the energies, we obtain for the critical thickness the value $\ell_{cr} \sim (\beta/k)^3\delta_{90}$.[1)]

In the case of iron this quantity is of the order of 10^3 cm, so that maximally branched structure cannot be realized in practice.

We have assumed so far that there is no external field. In the presence of an external field H_0 perpendicular to the plane of the plate, the branching becomes favored energywise if the plate surface is perpendicular to the easy axis. A model of the branched structure is shown in Fig. 2. All the formulas for this case are analogous to the preceding ones. They are obtained from formulas (1 - 3) by putting $\gamma = 0$ and making the substitution

$$k \to k[1 - (H_0/4\pi M)^2]\;.$$

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1) We were unable to take into account here the numerical factors, since the energy of the branched structure was not calculated for this case.