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PRODUCTION OF e^+e^- PAIRS IN AN ALTERNATING EXTERNAL FIELD

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Submitted 25 January 1971
ZhETF Pis. Red. 13, No. 5, 261 - 263 (5 March 1971)

Interaction of an electromagnetic field with a vacuum of charged particles leads to the appearance of nonlinear additions to the Lagrangian L of the electromagnetic field. In particular, there appears an imaginary part Im L, which determines the probability w of pair production by the external field. In the case of fields $\vec{E}$ and $\vec{H}$ constant in time (but alternating in space) it is possible to solve this problem exactly; this was done in [1] for scalar and spinor particles and in [2] for vector bosons with a gyromagnetic ratio g = 2. The following expression was obtained in these papers for the probability of pair production in a constant electric field E:

$$w = \frac{(2s + 1)\alpha E^2}{4\pi^2} \sum_{n=1}^{\infty} \frac{\beta_n}{n^2} \exp\left(-\frac{n\pi m^2}{eE}\right). \qquad (1)$$

Here $\hbar = c = 1$, $\alpha = e^2/\hbar c = 1/137$, s and m are the spin and mass of the particles produced by the field E, $\beta_n = (-1)^{n-1}$ for bosons, and $\beta_n = 1$ for fermions. The argument of the exponential in (1) is of the form nE_0/E, where $E_0 = \pi m^2 c^3/e\hbar = 4 \times 10^{16}$ V/cm (for electrons; since $E_0 \sim m^2$, the characteristic field intensity E_0 for other particles is even larger). We can therefore confine ourselves in the sum (1) to the first term with n = 1 for all the fields presently conceivable; this corresponds (as will be shown below) to the quasi-classical approximation.

We have obtained a generalization of (1) to the case of time-varying fields E(t). From the point of view of Dirac's theory, production of e^+e^- pairs corresponds to penetration of electrons with negative energy through a barrier of height $2mc^2$. In the quasiclassical approximation, the pair-production probability is

$$w \sim e^{-Q}, \qquad Q = 2\,\mathrm{Im}S \qquad (2)$$

(accurate to the pre-exponential factor), where S is the action acquired by the particle as it moves along the under-the-barrier trajectory. Such a motion, which is impossible in classical mechanisms, can be regarded as occurring for pure imaginary values of the "time" t (cf. [3, 4], where this method was developed for ionization of atoms by an alternating electric field). To find the argument Q of the exponential, it suffices to find the extremal trajectory (that minimizes Im S), which in this case is one-dimensional. Using the equation of motion $\dot{p} = eE(t)$ and the expression $L = -m(1 - \dot{x}^2)^{1/2} + eE(t)x$, we can represent the action S in the form

$$S(t) = px - \int_{t_0}^{t} \sqrt{p^2 + m^2}\, dt$$

(for the case when the field E(t) is homogeneous in space and is directed along the x axis). In the complex t plane, the action S(t) has branch points t_0, which are determined from the equation $p(t_0) = \pm im$. Finally,

$$Q = \oint (p^2 + m^2)^{1/2} dt, \tag{4}$$

where the contour C encloses the cut of the function $(p^2 + m^2)^{1/2}$ from $-t_0$ to t_0. To find t_0 and Q it is necessary to use the concrete form of the field E(t). Let us consider some examples.

1) In the case of a constant field E(t) = E, p(t) = eEt. Hence $t_0 = im/eE$ and

$$Q = \frac{\pi m^2}{eE} = \frac{E_0}{E}, \tag{5}$$

which coincides with the first term of the Schwinger formula [1].

2) For a field $E(t) = E \cos \omega t$ (corresponding to laser light), we have

$$p(t) = \frac{eE}{\omega} \sin \omega t, \quad t_0 = \frac{i}{\omega} \operatorname{Arsh} \frac{m\omega}{eE}, \tag{6}$$

and calculation of the integral (4) yields

$$Q = \frac{4m^2\sqrt{1+\gamma^2}}{eE\gamma^2} \left\{ K\left(\frac{\gamma}{\sqrt{1+\gamma^2}}\right) - E\left(\frac{\gamma}{\sqrt{1+\gamma^2}}\right) \right\}, \tag{7}$$

where K and E are complete elliptic integrals of the first and second kind, and γ is the adiabaticity parameter[1)]

$$\gamma = \frac{\omega}{\omega_t} = \frac{m\omega}{eE}. \tag{8}$$

Although $\omega \ll m$ and $E \ll E_0$, still γ can vary in a wide range, from $\gamma = 0$ (constant field) to $\gamma \gg 1$ (in this case the field E(t) has time to reverse its sign many times during the time of passage of the electron through the barrier). Formula (7) gives a continuous transition from one region to the other.

In the adiabatic region $\gamma \ll 1$ we have $Q = (E_0/E)(1 - \gamma^2/8)$ and (7) goes over into (5), while in the opposite case $\gamma \gg 1$ we have $Q = (4E_0/\pi E)(\ln \gamma/\gamma)$, i.e., Q decreases with increasing γ. Just as in the case of multiquantum ionization [5, 3], this leads to a sharp increase of the probability w. Expression (2) assumes in the region $\gamma \gg 1$ the form typical of N-th order perturbation theory

$$w \sim (E/E_1)^{2N}; \quad N = \frac{2m}{\omega}, \quad E_1 = \frac{m\omega}{e} = \frac{\omega}{\pi m} E_0 \tag{9}$$

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1) Here ω_t is the tunneling frequency. In an electric field E, the width of the barrier is $x_0 \sim mc^2/eE$ and the electron velocity is $\sim c$, whence $\omega_t = c/x_0$ (compare with the analogous determination of the parameter γ in the case of multiphoton ionization of atoms [5]).

(we note that $E_1 << E_0$). As seen from (8), the parameter γ increases together with the field frequency ω.

The conditions for the applicability of the quasiclassical approach are:

$$E << E_0 = \frac{\pi m^2}{e}, \quad \omega << m. \tag{10}$$

3) As the last example, let us consider a pulsed field in the form[2)] $E(t) = E(\cosh \omega t)^{-2}$. Here $t_0 = (i/\omega)\tan^{-1}\gamma$,

$$Q = \frac{2\pi m^2}{E(1 + \sqrt{1 + \gamma^2})} = \begin{cases} \frac{E_0}{E}\left(1 - \frac{1}{4}\gamma^2\right), & \text{for } \gamma << 1 \\ 2\pi \frac{m}{\omega}, & \text{for } \gamma >> 1 \end{cases}. \tag{11}$$

The main conclusion drawn from the foregoing examples is that the pair-production probability w increases strongly on going into the region $\gamma >> 1$ (at the same value of the field intensity E). The quasiclassical method does not require an exact solution of the Dirac equation (which is possible only in exceptional cases) and is therefore applicable to a large class of fields. It can be used to find not only the exponential exp(-Q) in the formula for w, but also the exact form of the pre-exponential factor, and also to take into account the influence of the magnetic field H. For lack of space, we cannot dwell on these questions here.

The author is grateful to A.M. Perelomov and M.V. Terent'ev for numerous useful discussions during the course of the work, and also to I.Yu. Kobzarev for a discussion of the results.

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COLD ELECTRONS IN SEMICONDUCTORS

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Submitted 25 January 1971
ZhETF Pis. Red. 13, No. 5, 264 - 266 (5 March 1971)

In homogeneous semiconductors the current always flows in the direction of the electric field. In inhomogeneous semiconductors, where the total current

[2)] We note that an exact solution of the Dirac equation was obtained recently for a field of this type; cf. [6].