(we note that $E_1 << E_0$). As seen from (8), the parameter γ increases together with the field frequency ω.

The conditions for the applicability of the quasiclassical approach are:

$$E << E_0 = \frac{\pi m^2}{e}, \quad \omega << m. \tag{10}$$

3) As the last example, let us consider a pulsed field in the form[2)] $E(t) = E(\cosh \omega t)^{-2}$. Here $t_0 = (i/\omega)\tan^{-1}\gamma$,

$$Q = \frac{2\pi m^2}{E(1 + \sqrt{1 + \gamma^2})} = \begin{cases} \frac{E_0}{E}\left(1 - \frac{1}{4}\gamma^2\right), & \text{for } \gamma << 1 \\ 2\pi \frac{m}{\omega}, & \text{for } \gamma >> 1 \end{cases}. \tag{11}$$

The main conclusion drawn from the foregoing examples is that the pair-production probability w increases strongly on going into the region $\gamma >> 1$ (at the same value of the field intensity E). The quasiclassical method does not require an exact solution of the Dirac equation (which is possible only in exceptional cases) and is therefore applicable to a large class of fields. It can be used to find not only the exponential exp(-Q) in the formula for w, but also the exact form of the pre-exponential factor, and also to take into account the influence of the magnetic field H. For lack of space, we cannot dwell on these questions here.

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COLD ELECTRONS IN SEMICONDUCTORS

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In homogeneous semiconductors the current always flows in the direction of the electric field. In inhomogeneous semiconductors, where the total current

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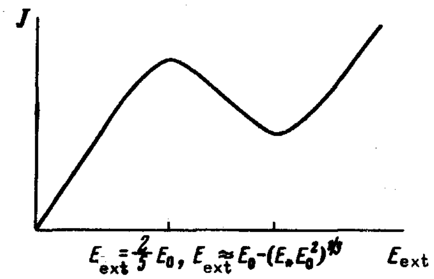
2) We note that an exact solution of the Dirac equation was obtained recently for a field of this type; cf. [6].



consists of the drift current and the diffusion current, the total current can flow either in the direction of the internal field, or in the opposite direction, depending on the polarity of the applied voltage. In the latter case, the field causes not heating but cooling of the electron (or hole) gas; this cooling can be quite appreciable if the internal field in the semiconductor is large.

To find the temperature T of the electron gas under stationary conditions (we consider, for concreteness, a donor semiconductor in which the donor concentration N_d is a function of x), it is necessary to equate the power P received by the electrons from the lattice to the power $\vec{J}\cdot\vec{E}$ given up to the electric field. The power is (cf., e.g., [1]).

$$P = C \frac{T - T_0}{\tau_e}, \tag{1}$$

where C is the specific heat of the electron gas, T_0 the lattice temperature, and τ_e the free-path time relative to collisions with energy loss.

If the lattice temperature is high enough, all the donors are ionized, and the characteristic distance L over which the impurity concentration varies is much larger than the Debye length L_D, we can write for the current J

$$J = \frac{e^2 N_d \tau_p}{m}\left(E - \frac{T}{e}\frac{1}{N_d}\frac{dN_d}{dx}\right), \tag{2}$$

where τ_p is the free-path time with respect to collisions with momentum loss. In writing down this equation, it was also assumed that the Einstein relation between the diffusion coefficient and the mobility remains in force at all fields E, and that $(dN_d/dx)/N_d$ = const.

The electron energy is usually scattered by collisions with the acoustic phonons, so that $\tau_e(T) = \tau_e(T_0)(T_0/T)^{1/2}$. On the other hand, momentum scattering can occur on both acoustic phonons and on ionized impurities. In the situation of interest to us, when the electron gas is cooled, scattering by impurities will predominate at low electron temperatures. Then $\tau_p(T) = \tau_p(T_0)(T/T_0)^{3/2}$.

Multiplying now (2) by E and equating to (1), we arrive at the following equation for the electron temperature

$$\theta^2 - \theta\left(\frac{E}{E_0} - \frac{E_*}{E}\right) - \frac{E_*}{E} = 0. \tag{3}$$

Here $\theta = T/T_0$; $E_0 = (T_0/e)(1/N_d)(dN_d/dx)$ is the internal field in the semiconductor at zero potential difference;

$$E_* = mCT_0/e^2 N_d \tau_e(T_0)\tau_p(T_0)E.$$

It is easy to verify that for an appreciable cooling of the electrons ($\theta \ll 1$) it is necessary to satisfy the condition $E_* \ll E_0$. If we write this condition in the form

$$eE_0 \frac{e\tau_p(T_0)}{m} E_0 >> \frac{ms^2}{\tau_\epsilon(T_0)} , \tag{4}$$

(s is the speed of sound), then we see readily that this condition amounts to a requirement that is natural in this case, namely that the energy given up by the electrons to the field at $T = T_0$ greatly exceed the energy the electrons receive from the lattice. It turns out that θ as a function of E has a minimum, which is reached (if (4) is satisfied) at the point

$$E = \left(\frac{1}{2} E_* E_0^2\right)^{1/3}. \tag{5}$$

The minimum electron temperature is $T_{min} \simeq T_0(E_*/E_0)^{1/3} << T_0$, with

$$\theta = \begin{cases} \dfrac{E}{E_0} + \dfrac{E_*(E_0 - E)}{E^2} & \text{for } E >> \left(\dfrac{1}{2} E_* E_0^2\right)^{1/3} . \\ \left(\dfrac{E_*}{E}\right)^{1/2} & \text{for } \left(\dfrac{1}{2} E_* E_0^2\right)^{1/3} >> E >> (E_* E_0)^{1/2} \end{cases} \tag{6}$$

It must be emphasized that when the field exerts a cooling effect the carrier temperature must have a minimum as a function of the external field regardless of the energy and momentum scattering mechanism. The reason is that if the external field E_{ext} is weak, the current J is small, and if E_{ext} is strong then the internal field $E = E_0 - E_{ext}$ is small. Consequently, the energy drawn from the electrons should be maximal (and consequently the electron energy minimal) at a certain intermediate value of E_{ext}.

Equations (6) and (2) can be used to determine the current-voltage characteristic of the semiconductor. Its form is shown in the figure. The most interesting feature of this characteristic is the presence of a decreasing branch. As is well known, this means that under the described conditions the electron gas is unstable against density fluctuations, and stationary current oscillations, similar to the Gunn effect, can be produced in it under certain conditions. The instability of cold electrons will be the subject of a separate study.

In conclusion, we present numerical estimates for the internal field E_0 in Ge. At room temperature, the speed of sound in Ge is of the order of 5×10^5 cm/sec and the mobility in scattering by phonons and impurities is $\sim 3 \times 10^3$ cm^2/V-sec. It follows then from (5) that $E_0 > 10^2$ V/cm. This leads to the limitation $L < 2.5 \times 10^{-4}$ cm; the Debye length is $L_d < 10^{-6}$ cm at $N_d > 10^{16}$ cm^{-3}.

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