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ROLE OF EXCITON-PHONON COMPLEXES IN THE INTERBAND MAGNETOOPTICAL ANOMALY

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Johnson and Larsen [1] observed an anomaly in the interband absorption of InSb in a strong field H when the electron cyclotron frequency ω_c approached the frequency ω of the polarized phonons. The singularity arising under these conditions in the energy spectrum of the polaron was calculated in [1, 2] and an interpretation of the observed effect was proposed on the basis of this singularity. It is indicated in [3] that this rearrangement of the spectrum corresponds to formation of electron-phonon complexes.

The interpretation of the absorption singularities at $\omega_c \simeq \omega$ on the basis of electron-phonon complexes is unconditionally correct if intraband transitions are involved (as, for example, in [4]). Since, however, transitions to the ground state of the exciton predominate in the intrinsic absorption in a strong magnetic field [5], failure to take the Coulomb interaction into account may qualitatively distort the result. We start here from the opposite approximation, i.e., we assume that the electron-phonon interaction is weak compared with the Coulomb interaction, and propose a theory of interband absorption at $\omega_c \simeq \omega$ in terms of exciton-phonon complexes.

We assume that the electron-phonon coupling is weak and that resonance $\omega_c \simeq \omega$ sets in only in the electron band; the hole-phonon interaction is therefore neglected. The magnetic field is assumed strong, i.e., $\omega_c \gg R$, where R is the exciton ionization potential at H = 0. The wave function of the system with zero total momentum in the one-phonon approximation is

$$\Psi = \Phi_{0,1,1} f_0(z)|0\rangle + \sum_q \Phi_{-q,0,1} f_q(z) b_q^+ |0\rangle \tag{1}$$

where the Oz axis is parallel to H. The function Φ_{q,n_1,n_2} describes the transverse motion of the free electron and hole in the Landau bands n_1 and n_2 with total momentum $q_\perp$ and the longitudinal motion of the center of gravity with momentum q_z. The functions of longitudinal relative motion f_0 and f_q are defined by the equations ($z = z_e - z_h$):

$$\left[-\frac{1}{2\mu}\frac{d^2}{dz^2} + U_{0,1,1}(z)\right] f_0 + \sum_q W_q(z) f_q = E f_0 \,,$$

$$\left[-\frac{1}{2\mu}\frac{d^2}{dz^2} + U_{-q_\perp,0,1}(z) - \omega_c + \omega + \frac{q_z^2}{2M}\right] f_q + W_q^*(z) f_0 = E f_q \,. \tag{2}$$

Here M and μ are the total and reduced masses, $U_{q_\perp,n_1,n_2}(z)$ is the Coulomb potential averaged over the transverse-motion functions that enter in Φ_{q,n_1,n_2},

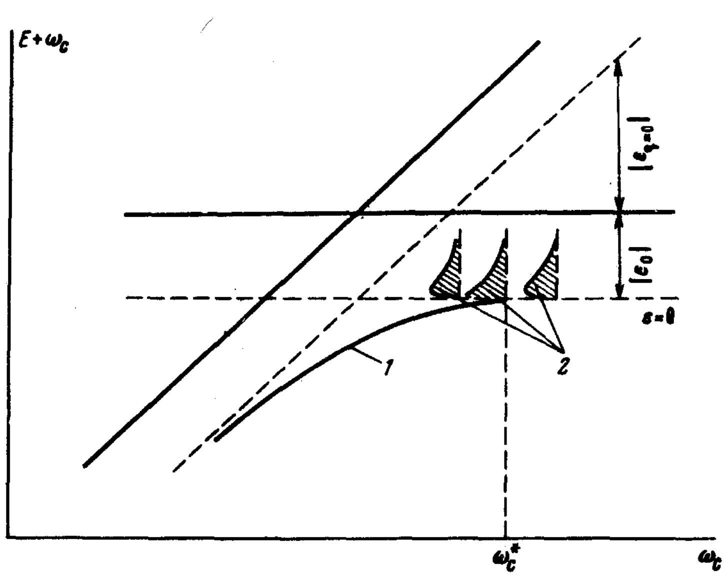
$$W_q(z) = W_q \exp\left(i\frac{m_h}{M} q_z z\right), \quad |W_q|^2 = \frac{4\pi a\omega}{q^2 V}\sqrt{\frac{\omega}{2m_e}} \exp\left\{-\frac{(\lambda q_\perp)^2}{2}\right\}\frac{(\lambda q_\perp)^2}{2} \tag{3}$$

is the matrix element of the electron-phonon interaction, taken between the functions $\Phi_{0,1,1}$ and $\Phi_{-q,0,1}$, and λ is the magnetic length. E is reckoned in (2) from the energy of the state in which the free electron and free hole are at the edges of the bands $n_1 = n_2 = 1$.

If we neglect the electron-phonon interaction, then the system (2) becomes decoupled, and its solutions f_0^0 and f_q^0 are the eigenfunctions of the one-dimensional Coulomb problem, with eigenvalues ε_0 and ε_q. We confine ourselves to consideration of the lower Coulomb levels. In the next approximation the energy spectrum is then determined by the equation

$$E - \epsilon_0 = \sum_q |B_q|^2 (E - \epsilon_q)^{-1}, \qquad B_q = (f_0^0 | W_q(z) | f_q^0). \tag{4}$$

This spectrum is shown in the figure. The solid lines correspond to position of the edge of the electron band $n_1 = 0$ plus the energy of the phonon ω and to the edge of the band $n_1 = 1$. The dashed lines correspond to the lowest Coulomb



Curve 1 - plot of the energy of the exciton-phonon complex vs. the cyclotron frequency. Curves 2 - distribution of the absorption in the continuous spectrum (the phonon has broken away from the exciton).

level of the excitons produced at the edges of these bands in the absence of interaction with the phonons. Curve 1, which lies at $\varepsilon \equiv E - \varepsilon_0 + \omega_c - \omega < 0$ represents the isolated root of Eq. (4) as a function of ω_c. In its left-hand side, it corresponds to an exciton produced at the bottom of the band $n_1 = 1$, and in the right-hand side to an exciton-phonon complex including the exciton at the edge of the band $n_1 = 0$. The line $\varepsilon = 0$ is the boundary of the continuous spectrum and at large E the phonon breaks away from the exciton.

If the optical transitions between the edges of the bands are allowed, then the transitions to the level $n_1 = 1$ go for the free carriers from the level $n_2 = 1$. Consequently the absorption I in the region of the $1 \to 1$ transition is proportional to the product of $|f_0(0)|^2$ by the density of state. It is given by the formula

$$I \sim \operatorname{Im}\Big(E - \epsilon_o - \sum_q |B_q|^2/(E - \epsilon_q)\Big)^{-1}, \quad E = E + i0. \tag{5}$$

The sums in (4) and (5) can be calculated approximately in the logarithmic approximation $L = \ln(a/\lambda)^2 >> 1$ (a is the Bohr radius), by using a quadratic expansion for ε_q in terms of $\vec{q}$ [6].

The end point ω_c^* of the one-particle spectrum at which the bound state of the complex vanishes, is

$$\omega_c^* \approx \omega\left[1 + \frac{\sqrt{\pi}}{2}\alpha\frac{a}{\lambda}\left(\frac{m_h}{Lm_e}\right)^{1/2}\right]. \tag{6}$$

The energy ε and the intensity I near ω_c^* are determined by the formulas

$$\frac{|\epsilon(\omega_c)|}{\omega} \approx \frac{m_e M}{m_h^2}\left[\frac{L}{a}\left(\frac{\lambda}{a}\right)^2\left(\frac{\omega_c - \omega_c^*}{\omega}\right)\right]^2, \quad I \sim \left(1 + \frac{\omega_c - \omega_c^*}{2\epsilon(\omega_c)}\right)^{-1} \tag{7}$$

i.e., $I \to 0$ as $\omega_c \to \omega_c^*$.

At $\varepsilon > 0$, the continuous absorption spectrum is strongly asymmetrical (curves 2 of the figure). Near ω_c^* the position of the maximum is determined by Formula (7); the absorption decreases to about one-half its value at $15\varepsilon_{max}$.

The weak dependences of ε and ε_{max} on ω_c near ω_c^* (like $(\omega_c - \omega_c^*)^2$) should become experimentally manifest by the presence of pinning. The criterion of a weak electron-phonon coupling is

$$\frac{\alpha}{L^{5/2}}\left(\frac{\omega}{2R}\right)^{3/2}\left(\frac{m_h}{\mu}\right)^{1/2} << 1; \tag{8}$$

one cannot count on its being well satisfied in real materials. Allowance for the Coulomb interaction is important in principle, however, since it changes the picture qualitatively in the region of the long-wave section of absorption, viz., according to the exciton model the width of the one-particle band is determined by extraneous mechanisms, whereas according to the model in which the Coulomb interaction is neglected the width results from the recoil of the hole [2].

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