Superfluid currents induced in He II by crossed electric and magnetic fields

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It is established that crossed fields should induce in helium II superfluid currents of velocity $v_s = \alpha E \times H/mc$ (α is the polarizability of the helium atom and M is its mass).

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The purpose of the present paper is to show that properties traditionally assumed to be possessed only by superconductors should be observed also in superfluid systems. We consider, for the sake of argument He^4 and examine the behavior of an He^4 atom in a combined electric and magnetic field. The electric field polarizes the atom and as a result the center of gravity $\mathbf{r}_1$ of the electron cloud does not coincide with the center of gravity $\mathbf{r}_2$ of the nucleus. If we denote by U the interaction energy of the electrons with the nucleus, then in the classical description the equations of motions obviously take the form:

$$m_1\ddot{\mathbf{r}}_1 = -\frac{\partial}{\partial \mathbf{r}_1} U(\mathbf{r}_1 - \mathbf{r}_2) - e\mathbf{E}(\mathbf{r}_1) - \frac{e}{c}\dot{\mathbf{r}}_1 \times \mathbf{H}(\mathbf{r}_1), \tag{1}$$

$$m_2\ddot{\mathbf{r}}_2 = -\frac{\partial}{\partial \mathbf{r}_2} U(\mathbf{r}_1 - \mathbf{r}_2) + eE(\mathbf{r}_2) + \frac{e}{c}\dot{\mathbf{r}}_2 \times \mathbf{H}(\mathbf{r}_2). \tag{2}$$

These equations describe both internal motions in the atom (i.e., the time variation of the difference $\mathbf{r} \equiv \mathbf{r}_2 - \mathbf{r}_1$) and the motion of the atom as a unit [i.e., the change of the quantity $\mathbf{R} \equiv m_1\mathbf{r}_1 + m_2\mathbf{r}_2/(m_1 + m_2)$]. If the frequencies of the fields **E** and **H** are much less than the characteristic frequencies of the internal motion, then it follows from (1) and (2) that **r** varies quasistatically, i.e.,

$$e\mathbf{r} = \alpha\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}\right). \tag{3}$$

Here α is the polarizability of the helium atom, and **E** and **H** are functions of the coordinate **R**. The appearance of the second term in the right-hand side of (3) is due to the fact that the Lorentz force also polarizes the atom. With the aid of (3) and Maxwell's equations it is easy to obtain from (1) and (2) the equation of motion of the mass center **R**. If we put $\dot{\mathbf{R}} = \mathbf{v}$ and $m_1 + m_2 = M$, then the equation takes the form

$$\frac{d}{dt} M\left(\mathbf{v} + \frac{\alpha\mathbf{H} \times \mathbf{E}}{mc}\right) = \alpha \nabla\left(\frac{E^2}{2} + \frac{\mathbf{v}(\mathbf{H} \times \mathbf{E})}{c}\right). \tag{4}$$

Let us examine this equation. Assume that a homogeneous field **E** is applied first, after which a homogeneous magnetic field **H** is turned on. Turning on the field **H**, by virtue of Maxwell's equations, produces inhomogeneous electric fields. These, however, are proportional to $1/c$, so that the right-hand side of (4) is proportional to $1/c^2$ and can be omitted. Integrating the remaining equations and setting the integration constant equal to zero (in accord with the fact that in the absence of fields the atom has zero velocity), we get

$$\mathbf{v} = a\,\frac{\mathbf{E}\times\mathbf{H}}{Mc} \equiv -\mathbf{u}\,. \tag{5}$$

The physical reason why a neutral body "gathers" a velocity $-\mathbf{u}$ is the following. In the switching stage, either the field **E** or the Lorentz force $(e/c)\,\dot{\mathbf{R}}\times\mathbf{H}$ is inhomogeneous. Therefore, although the forces acting on the gravity center of a negative charge $\mathbf{r}_1$ and on the gravity center $\mathbf{r}_2$ of a positive charge are of opposite sign, they do not cancel each other. The resultant force moves the atom as a whole, and its action, as seen from (5), does not depend on the rate at which the fields are turned on.

We have dealt so far with a single atom. It is quite obvious to ascertain what happens if the atom is some singled-out particle in a liquid. If the liquid is normal, then, in view of the presence of viscosity, the liquid velocity vanishes as $t\to\infty$. If, however, the liquid is superfluid, then even as $t\to\infty$ the liquid velocity is given by (5).

This statement can be proved more rigorously. The Lagrange function corresponding to Eq. (4) is

$$L = \frac{Mv^2}{2} + a\left(\frac{E^2}{2} + \frac{\mathbf{v}(\mathbf{H}\times\mathbf{E})}{c}\right) \tag{6}$$

since it is obvious that substitution of (6) in the Lagrange equation $(d/dt)(\partial L/\partial v_i) = (\partial L/\partial R_i)$ leads to (4). With the aid of the Lagrangian we can obtain in the usual manner the generalized momentum **p** and the system Hamiltonian $H(\mathbf{p},\mathbf{R})$

$$H \equiv \frac{\partial L}{\partial v_i}v_i - L = \frac{1}{2M}\left(\mathbf{p} - a\,\frac{\mathbf{H}\times\mathbf{E}}{c}\right)^2 - \frac{aE^2}{2} \tag{7}$$

The Hamiltonian (7) can be used to describe the condensate in He II, after first replacing in it the momentum **p** by the operator $-i\hbar\nabla$. We assume to this end (see, e.g.,[11,21]) that the behavior of the capacitor is determined by the wave function $\psi(\mathbf{r},t) = \psi_0(\mathbf{r}t)\exp[i\phi(\mathbf{r}t)]$, which satisfies the Schrödinger equation (μ is the chemical potential)

$$i\hbar\,\frac{\partial\psi}{\partial t} = (H + \mu)\,\psi\,. \tag{8}$$

The wave function ψ is so normalized that $M\psi_0^2=\rho_s$, where ρ_s is the mass density of the superfluid component. Separating the imaginary part in Eq. (8), we obtain the continuity equation

$$\dot{\rho}_s + \operatorname{div} \rho_s \mathbf{v}_s = 0, \tag{9}$$

where the velocity of the superfluid component is

$$\mathbf{v}_s = \frac{\hbar}{M} \nabla \phi - \mathbf{u}. \tag{10}$$

The real part of (9) is given by

$$\frac{\partial}{\partial t} M (\mathbf{v}_s + \mathbf{u}) = - \nabla \left(\mu + \frac{M v_s^2}{2} - \alpha \frac{E^2}{2} \right). \tag{11}$$

We have left out of the right-hand side of this equation the term $(\hbar^2/2M)\nabla[(\nabla^2\sqrt{\rho_s}/\sqrt{\rho_s})]$, which is significant only if superfluid density varies very rapidly as a function of the coordinates.

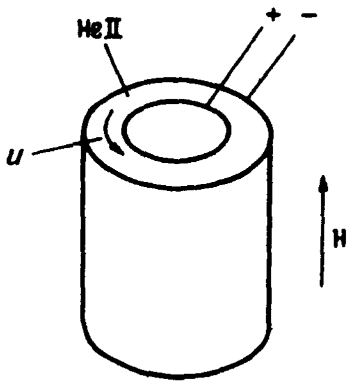

FIG. 1.

We shall show that in the case of a metallic vessel of arbitrary shape (see Fig. 1), neglecting the compressibility of the helium, $\mathbf{v}_s = -\mathbf{u}$ is a solution of the system (9)–(11). In fact, for an incompressible liquid $\mathbf{v}_s$ should satisfy the equation $\operatorname{div} \mathbf{v}_s = 0$ with boundary condition $\mathbf{v}_{s\perp} = 0$, where $\mathbf{v}_{s\perp}$ is the velocity component normal to the surface. If $\mathbf{v}_s = -\mathbf{u}$, then the conditions $\operatorname{div} \mathbf{u} = 0$ and $\mathbf{u}_\perp = 0$ should obviously be satisfied. It is easy to find with the aid of Maxwell's equations that $\operatorname{div} \mathbf{u} \equiv (\alpha/Mc)\operatorname{div} \mathbf{H} \times \mathbf{E} = (\alpha/Mc)(1/rc)(\partial/\partial t)(E^2 + H^2)$. Consequently, after the fields reach their stationary values we have $\operatorname{div} \mathbf{u} = 0$. The condition $\mathbf{u}_\perp = 0$ on the surface of a metallic vessel is also satisfied, since $\mathbf{E}$ is directed normal to the surface of the vessel, and $\mathbf{uE} = (c/Mc)(\mathbf{H} \times \mathbf{E})\mathbf{E} \equiv 0$. Thus, we have again arrived at the result (5). Allowance for the compressibility effects leads to the appearance in the inhomogeneous cases of weak gradients of the phase $\nabla\phi$ [see (10)].

Expression (5) admits of a simple interpretation , by which this result can be obtained in a perfectly clear manner. The helium II polarized by the homogeneous electric field is, in a certain sense, the analog of a superconducting layered crystal in

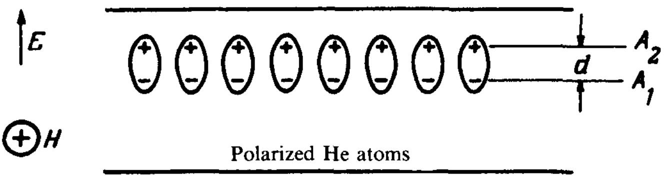

FIG. 2.

which layers having n- and p-type conductivity alternate (see Fig. 2). When such a crystal is placed in a magnetic field, a Meissner current should be produced in each layer. Since the particles of the neighboring layers are pairwise coupled, the total current is determined by the difference of the Meissner currents in the upper and lower layers (see[3,4]). The velocity of the liquid should therefore be [see (2)]

$$v_s = \frac{e(A_1 - A_2)}{Mc} = \frac{edH}{Mc} = \frac{\alpha EH}{Mc}, \tag{12}$$

which coincides with expression (5).

We present a numerical estimate of the velocity u. The polarizability of the helium atom is $\alpha \approx 2\times 10^{-25}$ cm^3 and the mass is $M \approx 10^{-23}$g. Therefore in fields $H \sim 10^5$ G and $E \sim 10^5$ V/cm we have $u = 2\times 10^{-4}$ cm/sec. This is a low but quite measurable quantity.

The described experiment in crossed fields is not the only situation in which the predicted effect can arise. If charged particles are introduced into helium II, then turning on the magnetic field should produce around each of them a spindle-shaped vortex whose velocity is given by (5). The projection of the vortex velocity on a plane normal to the magnetic field is

$$v_{sr} = \frac{\alpha H}{Mc}\frac{Q}{R^2}\sin\theta .$$

Here Q is the particle charge,.R is the distance from the charge to the point in question, and θ the angle reckoned from the magnetic field. One of the possible ways of detecting the predicted vortices is to produce a stream of the superfluid component. The vortices (together with the charges) will then be acted on by a Magnus force, and the charge should be set in motion. This and other questions are considered in detail in our paper submitted to Fizika Nizikikh Temperatur (Low Temperature Physics).

We note in conclusion that the described effects should hold also in other superfluid systems, particularly superfluid He^3.

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[1]V.L. Ginzburg and A.A. Sobyanin, Usp, Fiz. Nauk **120**, 153 (1976) [Sov. Phys. USP. **19**, 773 (1976)].
[2]R. Feynmann, Statistical Mechanics, Benjamin, 1972.
[3]S.I. Shevchenko, Fiz. Nizk. Tem. **2**, 505 (1976) [Sov. J. Low Temp. **2**, 251 (1976)].
[4]S.I. Shevchenko, *ibid*. **3**, 605 (1977) [**3**, 293 (1977)].