Conservation of electric charge

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The hypothesis that electrons are unstable is discussed. It is shown that the longitudinal photons make it impossible to construct a self-consistent theory of charge nonconservation.

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1. The question of the restrictions on the possible nonconservation of electron charge, imposed by the circumstance that the photon is practically massless, was raised in[1]. It was shown with a simple example that spontaneous violation of the electric-charge conservation is impossible within the framework of a renormalizable theory, and the hypothesis was advanced that in a nonrenormalizable theory the virtual photons can in practice reduce to zero the effective constant that characterizes the charge nonconservation. In the present article we explain why spontaneous violation of charge conservation is impossible, and demonstrate the mechanism of "self-healing" of quantum electrodynamics.

We consider for the sake of argument the interaction of electrons, neutrinos, and the hypothetical scalar mesons χ:

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$$g(\bar{e}\nu + \bar{\nu}e)\chi\ . \tag{1}$$

Our conclusions, however, are valid for other types of charge-nonconserving interactions as well as for other particles. Thus, for example, (1) can contain several neutron particles in lieu of the neutrino, a proton in lieu of the electron, etc.

2. Let us try to obtain the interaction (1) via spontaneous violation from the Lagrangian

$$L = |(\partial_\mu - ieA_\mu)\phi|^2 + \sqrt{2}g(\bar{e}\nu\phi^+ + \bar{\nu}e\phi) - \frac{\lambda^2}{8}(2|\phi|^2 - \eta^2)^2 + L_o . \tag{2}$$

Here ϕ is a scalar charged field whose interaction with e and ν conserves the charge, and L_0 is the usual Larangian that describes the photons, electrons, and neutrinos. The Hamiltonian corresponding to (2) has a minimum at $\phi = \eta/\sqrt{2}$. As a result, the field ϕ acquires a vacuum mean value $\langle\phi\rangle = \eta/\sqrt{2}$, and in place of one charged field we obtain two neutral fields: the longitudinal component A_3 of the photon field and the neutral scalar field χ ($\sqrt{2}\phi = \eta + iA_3 + \chi$). This gives rise to terms that do not conserve the electric charge, $g(\bar{e}\nu + \bar{\nu}e)(\chi + \eta)$, and the photon and χ meson acquire masses: $m_\gamma = e\eta,\ m_\chi = \lambda\eta$. If perturbation theory is to be valid and renormalizability is to take place, it is necessary that the self-action of the field ϕ be small: $\lambda \lesssim 1$. But this means that m_χ can subtantially exceed m_γ. The observations,[2] on the other hand, give a very low limit for m_γ:$1/m_\gamma \gtrsim 10^{22}$ cm. From this it follows immediately that for all frequencies $\omega \gg m_\gamma$ (and it is precisely with such frequencies that we deal in experiment) we can neglect the term with η in the Lagrangian (2). The splitting of the field ϕ into two neutral fields (A_3 and χ) occurs only at distances exceeding $\omega(m_\gamma - m_\chi)^{-2}$. In experiment, on the other hand, we deal not with two neutral fields, but with a fixed superposition of these fields, which describes a charged and practically massless meson ϕ. The existence of such a meson is completely excluded by the experimental data aimed to verify quantum electrodynamics. To make the ϕ-field mass acceptably large ($\gtrsim 1$ GeV) the constant would have to be so large ($\lambda \gtrsim 10^{35}$), that there could be no talk of perturbation theory at all.

3. We consider now a nonrenormalizable Lagrangian that patently does not conserve charge:

$$L = \frac{1}{2}\left(\frac{\partial\chi}{\partial x_\mu}\right)^2 + g(\bar{\nu}e + \bar{e}\nu)\chi + \frac{1}{2}m_\gamma^2 A_\mu^2 + L_o . \tag{3}$$

Here χ is a neutral massless scalar field. (Nothing changes in the subsequent arguments if we assume that the field χ has nonzero mass or nonzero spin. In particular, the role of χ could be assumed by the electromagnetic field: $\chi = \gamma$. In place of χ it is also possible to consider a neutral multiparticle state, for example $\chi = \nu\bar{\nu}$. If $m_\chi > m_e$, then the electron would be stable and the subsequent arguments would pertain to the $\chi \to e\nu$ decay.) The term $\frac{1}{2}m_\gamma^2 A_\mu^2$ in the Lagrangian (3) describes the photon mass, which is inevitable in the case of charge conservation. The emission and absorption of real and virtual photons with longitudinal polarization leads to the appearance of singularities as $m_\gamma \to 0$. These singularities in the amplitude are proportional to

$(\omega/m_\gamma)^k$, where k is the number of longitudinal-photon emission and absorption vertices, so that at $\omega \gg m_\gamma$ it is necessary to sum these terms. It is convenient to carry out this summation by changing from the Lagrangian of massive photons to a sum of two Lagrangians, the first of which describes ordinary massless photons γ, and the second massless scalar mesons which we shall designate σ. (The number of degrees of freedom remains the same here: 3, 2, and 1 for Proca, Maxwell, and Klein-Gordon fields, respectively). This subdivision is valid if the frequencies (momenta) of the real and virtual photons are large compared with m_γ. Introduction of the field σ is affected by formally substituting $A_\mu \to \partial_\mu \sigma / m_\gamma$ in the Lagrangian. (The massive-photon propagator in the Feynman diagrams $i(\delta_{\mu\nu} - k_\mu k_\nu / m_\gamma^2)/(k^2 - m_\gamma^2)$ is then replaced by a sum of two propagators $i\delta_{\mu\nu}/k^2$ and $+ ik_\mu k_\nu / m_\gamma^2 k^2$). Those terms of the Lagrangian which contain the σ field take the form

$$\frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 - ie\,\bar{e}\gamma_\mu \frac{\partial\sigma}{\partial x_\mu} e\,.$$

If we now carry out a gauge transformation of the electron field e, then the second term can be made to vanish, the gauge-invariant term $g\chi(\bar{e}\nu + \bar{\nu}e)$ yields then an exponential interaction, and we get

$$\frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 + g\chi\left(\bar{\nu}e\exp(ie\sigma/m_\gamma) + \bar{e}\nu\exp(-ie\sigma/m_\gamma)\right). \tag{4}$$

Thus, emission and absorption of longitudinal photons occurs at the points where the charge vanishes (the electron "throws off" the "jacket" of longitudinal photons). As we shall presently see, it is impossible to throw off this jacket instantaneously, so that the effective constant g vanishes.

4. To calculate the renormalization factor g, we expand $\exp(ie\sigma/m_\gamma)$ in powers and close $2n$ lines in the Nth term, leaving k free ends ($N = 2n + k$, see Fig. 1)

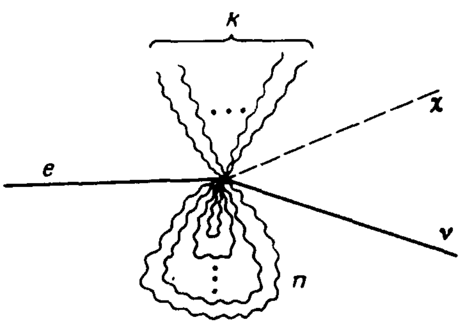

FIG. 1.

The term with indices n and k in the effective Lagrangian that violates charge conservation will take the form

$$\tilde{L}_{n,k} = \frac{g}{k!}\left(\frac{ie\sigma}{m_\gamma}\right)^k \frac{(J/2)^n}{n!}, \tag{5}$$

where J is the contribution of one loop:

$$J = \frac{e^2 i^3}{m_\gamma^2 (2\pi)^4} \int \frac{d^4 k}{k^2 + i0} = -\frac{4\pi a}{m_\gamma^2} \frac{4\pi^2}{(2\pi)^4} \int_0^\Lambda \frac{|\mathbf{k}|^2 d|\mathbf{k}|}{\mathbf{k}} = -\frac{a}{2\pi} \frac{\Lambda^2}{m_\gamma^2}. \quad (6)$$

Owing to the divergence, we had to introduce a cutoff parameter. If we sum now over n from 0 to ∞, we obtain for a given k

$$\widetilde{L}_k = \frac{g}{k!} \left(\frac{ie\sigma}{m_\gamma} \right)^k e^{-\frac{a}{4\pi} \frac{\Lambda^2}{m_\gamma^2}} . \quad (7)$$

The effective charge-nonconservation constant is therefore

$$\widetilde{g} = g e^{-a\Lambda^2/4\pi m_\gamma^2} . \quad (8)$$

Let us explain the meaning of the factors in (5). The k lines can be chosen from among the N lines in $N!/k!(N-k)! = N!/k!2n!$ ways. Next, $2n$ lines can be paired in $(2n-1)!! = 2n!/2^n n!$ ways. Gathering the factors together, we have

$$\frac{1}{N!} \frac{N!}{k!2n!} \frac{2n!}{2^n n!} = \frac{1}{k!\, n!\, 2^n} . \quad (9)$$

5. We calculate now the probability Γ_k of emission of k longitudinal photons ("σ quanta") in the process $e \to \nu\chi + k\gamma$:

$$\Gamma_k = \frac{\widetilde{g}^2 m_e}{16\pi} \left(\frac{a m_e^2}{4\pi m_\gamma^2} \right)^k \frac{1}{(k!)^2 (k+1)!} . \quad (10)$$

The factor $k!$ accounts here for the identity of the photons, while the factor $k!(k+1)!$ stems from the phase space of the $k+2$ particles (with allowance for the neutrino and the χ meson). Summing over k, we have

$$\Gamma = \sum_k \Gamma_k \approx \widetilde{g}^2 m_e e^{3\left(\frac{a m_e^2}{4\pi m_\gamma^2} \right)^{1/3}} , \quad (11)$$

This formula, derived in[1], has exponential accuracy. The argument of the exponential contains the characteristic number $\bar{k}$ of photons emitted in the decay. If $1/m_\gamma = 10^{22}$ cm, then $\bar{k} \approx 10^{21}$ and the wavelength of each photon is of the order of 10^{10} cm. If the constant g were not exponentially small, the electron would dccay almost instantaneously. If we assume that $\Lambda \gg m_e$, then the electron, as follows from (11) and (8), is practically stable at any realistic value of g (for example, at g of the order of the constant of the ultraweak CP-odd interaction, see[3] and also[4]). Other effect that can arise in principle at $g \neq 0$, such as a nonzero electric charge of the neutrino or of the hydrogen atom, and others, will also be vanishingly small.

6. The exponential vanishing of g is the result of mutual cancellation of very large

terms. This cancellation would be violated if the cutoff limits in the individual loops on Fig. 1 were rigorously identical. We have no relativistic-cutoff model that yields finite values of Λ. The reason is that the charge and mass are quantized, and the electron cannot be made to vanish by a series of radiative transitions in each of which the probability of longitudinal-photon emission is small because of the small energy release or the small change in charge. Under these conditions it seems natural to assume that the emission of the longitudinal photons takes place in the local limit, and to eliminate $\Lambda \to \infty$. The longitudinal photons then "produce such a crush" that they cannot be themselves emitted and do not permit other particles (neutrinos) to be emitted, so that the conservation is fully restored.

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