Concerning one scale-invariant variable for the description of hadron-hadron interactions

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A new scale-invariant parametrization is proposed for single-particle inclusive distributions in hadron-hadron interactions. The variables $\xi^{\pm}$ introduced for this purpose go over in the fragmentation regions of the interacting particles at large primary energies into the well known Feynman variable x.

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A substantial role is played in the analysis of experimental data on single-particle inclusive distributions $E\,(\partial\sigma/\partial\mathbf{p})$ by the choice of the kinematic variables on which this distribution depends. The Lorentz-invariant parametrization proposed in this paper for the inclusive differential cross section in the limit of high energies in the fragmentation region (using the accepted terminology) of the interacting particles goes over into the known Feynman parametrization in the variable x.

We consider an aribtrary 4-vector p_μ $(p_0,\mathbf{p})$ and introduce for it the "light-front" variables[1]

$$p_\pm = p_0 \pm p_3 . \tag{1}$$

We note that if the 4-momentum p_μ lies on the mass shell, then combinations of $p_\pm$ and $\mathbf{p}_\perp$ [where $p_\perp = (p_1,p_2)$] specify the horispheric coordinate system on the corresponding hyperboloid.[2,3]

We make up from the 4-momenta p^a, p^b, p^c of the inclusive reaction $a+b\to c+X$ the scale-invariant combinations[1]

$$\xi^\pm = \pm \frac{p^c_\pm}{p^a_\pm + p^b_\pm} \tag{2}$$

(we take the z axis to be the collision axis, i.e., $p_z = p_3 = p_L$).

It is easily seen that the variables $\xi^\pm$ are Lorentz-invariant to transformations of the reference frame along the collision axis. It is of interest to note that combinations of variables close to (1) arise when scale transformations are considered in the theory with fundamental length.[6] In the analysis of the properties of inclusive distributions it may be quite useful to carry out a generalized Fourier analysis[3,7-9] on the hyperboloid that determines the mass shell of the particle.

In the case of high energies in the c.m.s. of the colliding particles, for particles c whose momenta satisfy the condition $|p^{c*}_L| \gg m^c_\perp$ (the fragmentation region according to the standard terminology), the variables go over into the known Feynman variable $x = 2p^*_L/\sqrt{s}$. In terms of the variables $(\xi^\pm,\mathbf{p}_\perp)$, the invariant differential cross section for the inclusive production of the particle c is of the form

$$E^c \frac{d\sigma}{\partial \mathbf{p}^c} = \frac{\xi^\pm}{\pi} \frac{\partial \sigma}{\partial \xi^\pm \partial p_\perp^{c\,2}} = f(\xi^\pm, p_\perp^{c\,2}, s). \tag{3}$$

The hypothesis of scale invariance in the variables $\xi^\pm$ is written as follows:

$$E^c \frac{\partial \sigma}{\partial \mathbf{p}^c} = \begin{cases} f_1(\xi^+, p_\perp^c) & \text{for particles with } p^*_L > 0 \\ f_2(\xi^-, p_\perp^c) & \text{for particles with } p^*_L < 0 \end{cases} . \tag{4}$$

In the case of interaction of identical particles we have $f_1 = f_2$.

Since the variables $\xi^\pm$ depend both on the transverse momentum and on the mass of the singled-out particle, we can expect that it will help subdivide more distinctly the phase space into the fragmentation and central regions for particles with different masses.

Preliminary results of the experimental investigation of inclusive reactions on the basis of the proposed new variables were obtained for $\pi^- p$ interactions. We used a simplified variant of the variable ξ:

$$\xi = \frac{p^c_+}{p^a_+} \frac{E^c + p^c_L}{E^a + p^a_L} . \tag{5}$$

We note that the variable ξ, unlike the variables $\xi^{\pm}$, is not symmetric about the point $x = 0(p_L^* = 0)$ and is suitable only for the investigation of the scale-invariant properties of the single-particle spectra in the beam fragmentation region.

Figure 1 shows the experimental values of the normalized differential cross section $(\xi/\sigma_{\text{inel}})(d\sigma/d\xi)$ for inclusively produced $\pi^{\pm}$ mesons in $\pi^- p$ interactions at beam momenta 5 and 40 GeV/c. The data at 5 GeV/c were obtained from stereo photo-

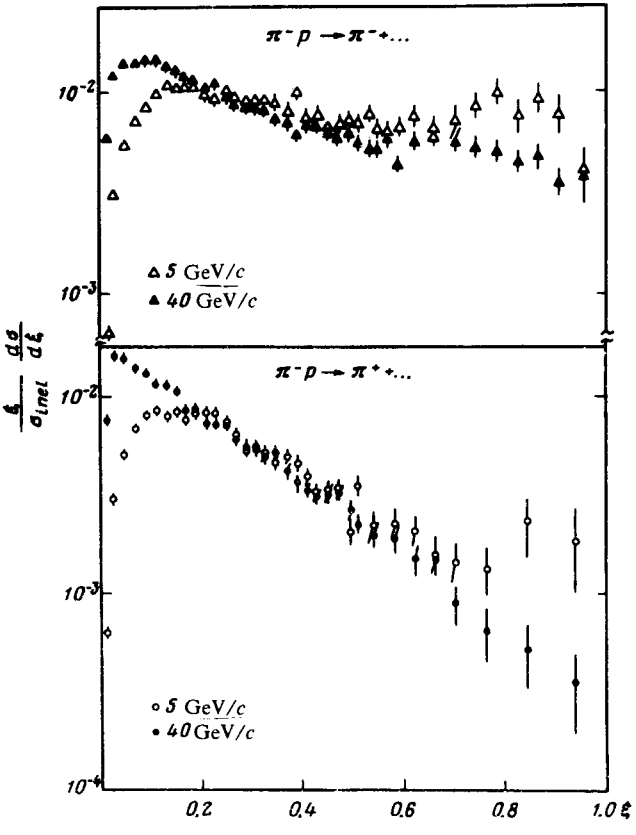

FIG. 1. Inclusive distributions $(\xi/\sigma_{\text{inel}})(d\sigma/d\xi)$ of $\pi^{\pm}$ mesons in $\pi^- p$ collisions at 5 GeV/c(Δ) and 40 GeV/c(▲).

graphs from the 1-meter hydrogen bubble chamber of the High-Energy Laboratory of the Joint Institute for Nuclear Resarch,[10] and the data at 40 GeV/c were obtained with the 2-meter propane bubble chamber of the same Laboratory.[11]

For greater clarity, Fig. 2 shows the experimental values of the ratio

$$R = \frac{((\xi/\sigma_{\text{inel}})\partial\sigma/\partial\xi)5/\text{GeV}/c}{((\xi/\sigma_{\text{inel}})\partial\sigma/\partial\xi)40/\text{GeV}/c}$$

as functions of ξ.

We see that in the range $0.2 \lesssim \xi \lesssim 0.7$, corresponding approximately to the fragmentation region of the beam particle, an approximate equality of the single-particle distributions $(\xi/\sigma_{\text{inel}})(d\sigma/d\xi)$ is observed ($R \approx 1$). In the region $\xi \gtrsim 0.7$ the errors due

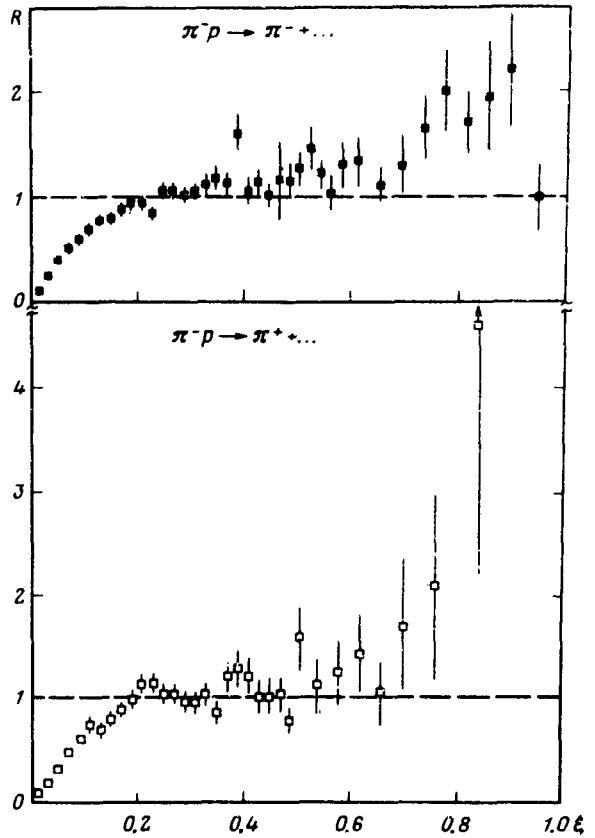

FIG. 2. The ratio $R = [(\xi/\sigma_{\text{inel}})(d\sigma/d\xi)]_{5\ \text{GeV}/c} / [(\xi/\sigma_{\text{inel}})(d\sigma/d\xi)]_{40\ \text{GeV}/c}$ as a function of the variable ξ.

to the inadequate statistics are quite large, so that no definite conclusions can be drawn.

Statistically well founded experimental data for various inclusive reaction in a wide range of interaction energies will lead to more definite conclusions concerning the scale-invariance of the distributions in the variable ξ. We are planning a detailed analysis of the corresponding data. We shall also analyze data on semi-inclusive reactions.

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[1]Variables of similar type proved to be quite convenient for the study of relativistic states of systems.[4,5]

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