Stabilizing influence of plasma flow on dissipative tearing instability

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Tearing instability of a current sheath (CS) is considered in the MHD approximation with account taken of the spreading of the plasma over the sheath. Such a flow can lead to a change in the instability threshold, thereby exerting a strong stabilizing influence.

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1. Tearing instability (TI) is presently being intensively investigated in connection with various applications.[1-3] According to the results of the theory, the CS in a plasma is practically always unstable. At the same time, laboratory investigations[4] and the interpretation of observations in outer space show that the current sheaths exist for prolonged times. Without dwelling on the various attempts to determine the stabilization mechanisms,[5-7] we note here an effect that can explain the difference between the theoretical conclusions and the observations.[1)]

In Ref. 1 they consider a setup with a static configuration. The actually existing current sheaths constitute inhomogeneous streams of plasma that flows into the sheath over a broad surface and exits through narrow ends. We demonstrate below, in the MHD formulation of the problem, the possibility of stabilizing the TI of a layer of finite thickness.

2. Following Ref. 1, we consider a CS lying in the plane $y=0$, but assume on the contrary that the plasma flows inhomogeneously over the sheath

$$v_x = hx, \qquad h > 0. \tag{1}$$

We are considering here conditions when the rate of inflow is much greater than the rate of outflow from the sheath. It can be shown that in the limit of high conductivity σ, such that $S=\tau_R/\tau_A \gg 1(\tau_R=4\pi\sigma L^2/c^2;\ \tau_A=v_A/L)$, the normal component of the velocity is of no significance to the investigated instability in our formulation of the problem (see Ref. 6).

We linearize the MHD equations about the initial state:

$$\mathbf{B} = B_\infty \mathrm{th}(y/L)\,\mathbf{e}_x + \mathbf{B}_1, \quad \mathbf{v} = hx\,\mathbf{e}_x + \mathbf{v}_1.$$

We express the dependence of the perturbations on the coordinate x and on the time t in the form

$$\left\{\begin{matrix} \mathbf{B}_1 \\ \mathbf{v}_1 \end{matrix}\right\} \sim \exp\{ik(t)x + \int^t \gamma(k(t'), t')\,dt'\}. \tag{2}$$

Here $k(t)$ is the wave number, which is a function of the time. If we choose $k(t)=k_0 \exp(-\int h dt)$, then we can obtain for the perturbations of the magnetic field B_{1y} and of the velocity v_{1y} the equations

$$(\gamma + h)B_{1y} = ikB_\infty \operatorname{th}(y/L)v_{1y} + \frac{c^2}{4\pi\sigma}(B''_{1y} - k^2 B_{1y}), \tag{3}$$

$$(\gamma + 2h)\,v''_{1y} - k^2\gamma v_{1y} = i\frac{kB_\infty \operatorname{th}(y/L)}{4\pi\rho_0}\left(B_{1y} - \left(k^2 - \frac{2}{L^2 \operatorname{ch}^2(y/L)}\right)B_{1y}\right). \tag{4}$$

The prime denotes differentiation with respect to y. Just as in Ref. 1, to solve Eqs. (3) and (4) we subdivide the CS into two regions, external, where the freezing-in conditions are satisfied and the motions can be regarded as adiabatically slow, and internal, where reclosing of the force lines takes place. Matching together the solutions on the boundary, we obtain the dispersion equation

$$1 - k^2L^2 = \frac{(\gamma + h)(\gamma + 2h)^{1/4}\, 2\pi\Gamma(3/4)(kL)^{1/2}\, \tau_R^2}{S^{1/2}\,\Gamma(1/4)}. \tag{5}$$

The instability threshold $k=k^*$ is obtained by putting $y=0$. If $k<k^*$, an instability takes place with the growth rate obtained in Ref. 1. At $S \gg 1$ and $h\tau_R > 1$ we have

$$k^* \approx \frac{S}{\sqrt{2}\,L(h\tau_R)^{5/2}}\left(\frac{\Gamma(1/4)}{\Gamma(3/4)\,2\pi}\right)^2. \tag{6}$$

Estimating $h \approx v_A/b$ and $kL \approx L/b$ (b is the width of the CS), we obtain the condition for the CS stability: $b < LS^{3/7}$, when the perturbations attenuate with a decrement $\gamma = -h$. ($S \approx 10^{10}$ under the conditions of the solar atmosphere.)[2]

3. The condition for the elimination of the instability can be obtained by comparing the values of the growth increment γ_0, calculated without allowance for the spreading flow, with the value of h. If $\gamma_0 < h$, then the sheath is stable. It is obvious that estimates of this kind are valid in a more general case. Let us apply them to the nonlinear stage of the tearing instability, when one or several null lines of the magnetic field are produced in the current sheaths. Let the nonlinear increment be equal to γ_n. In the vicinity of lines of type X, secondary flows are produced, for which

$$v_g l \approx v_x \delta, \tag{7}$$

$l \sim \pi/k$ is the characteristic dimension along the x axis, δ is the characteristic dimension along y. We estimate v_y at $\gamma_n \delta$. The decrement corresponding to the spreading of the plasma over the layer is of the order of $h_n \approx v_x/l$. It follows from (7) that $h_n \approx \gamma_n$, i.e., the secondary CS are at the stability limit.[3)]

It appears that for the tearing of a stabilized CS and for a more effective joining of the force lines it is necessary to resort to the anomalous-resistance mechanisms.[5,6]

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[1)]It should be noted that the stabilization of the normal component of the magnetic field, which is discussed in the literature, does not take place in the MHD regime. In the case of a thin sheath of a cold collisionless plasma, as shown in Ref. 9, the spreading of the plasma over the sheat ensures effective stabilization.

[2)]The relation $b < LS^{3/7}$ was obtained in Ref. 9 from qualitative considerations.

[3)]See in this connection Ref. 8, where it was shown on the basis of numerical simulation that the nonlinear stage of the tearing instability is accompanied by a singular flow with a current sheath whose dimensions are smaller than the initial one.

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