Nonlinearity of single-photon absorption in a superconductor near the threshold

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Single-photon absorption of the electromagnetic field in a pure superconductor at finite amplitudes is considered. It is shown that near the threshold the absorption becomes nonlinear at $H \gtrsim H_c \kappa^{1/2}[(\omega - 2\Delta)/\Delta]^{3/4}$, and that the nonlinearity leads to the appearance of the minimum on the amplitude characteristics and to a smoothing of the kink on the frequency characteristics at the threshold point.

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The process of single-photon production of electron-hole pairs in a superconductor by an alternating electromagnetic field has, as is well known, a threshold at the frequency $\omega = 2\Delta$. The absorption of the electromagnetic energy, connected with this process and occurring at $\omega < 2\Delta$, was considered many times within the framework of the theory of the linear response.[1] The present communication is devoted to absorption at finite amplitudes. It is shown that in pure superconductors near the threshold $\Omega = \omega - 2\Delta \ll \Delta$, single-photon absorption becomes nonlinear at an anomalously low ($H \ll H_c$) amplitude of the alternating field:

$$H \gtrsim H_c \kappa^{1/2}\left(\frac{\Omega}{\Delta}\right)^{3/4}, \qquad \kappa = \frac{\delta\Delta}{v_F}. \tag{1}$$

The reason of so early a nonlinearity is the strong disequilibrium of the pairs that are produced in the near-threshold region and are responsible for the single-photon absorption. In accordance with the conservation law $\epsilon + \epsilon' - w = 0$, the produced pairs are concentrated in the low-energy region $\epsilon - \Delta \sim (\Delta\Omega)^{1/2}$, and are therefore very sensitive to external action.[1)]

Under conditions of anomalous skin effect, the process of dissipation of electromagnetic energy can be divided into two stages. In the first stage, the electrons elastically scattered by the skin layer draw energy from the field, while in the second stage they transfer the energy in the interior of the superconductor to the phonon thermostat. After thermalization, the electrons again return to the skin layer. Calculation of the evolution of the electron distribution on going through the skin layer of a superconductor with a specularly reflecting boundary is based on the kinetic equation for the density matrix $\hat{\gamma}(x,x',t)$[2]:

$$i\,\dot{\hat{\gamma}} = [\sigma_z \eta v_F \hat{p} + \sigma_x \Delta + v_F p_s \cos\phi,\ \hat{\gamma}], \tag{2}$$

with an initial condition in the form of an equilibrium density matrix prior to the scattering (absence of heating), where η is the glancing angle and ϕ is the azimuthal angle of the electron trajectory. A solution of (2) at finite amplitude of the applied field can be obtained because of the large de Broglie wavelength of the low-lying excitations in a model with a pointlike potential $p_S(x,t) \to u \cos \omega t \delta(x)$. In this model the calculation of the energy imparted by the field to the electrons reduces to finding a function of discrete argument S_n, satisfying the recurrence relation

$$S_{\pm n} = - \frac{u_{\pm n}}{1 + u_{\pm n} S_{\pm(n+1)}} \; ; \qquad u_n = iB\left(\frac{\epsilon - \omega n + \Delta}{\epsilon - \omega n - \Delta}\right)^{\sigma/2} ;$$

$$B = \frac{\sigma' u \cos\phi}{\eta} ; \qquad \sigma, \sigma' = \pm 1 . \tag{3}$$

The energy absorbed by the electrons, averaged over the period, is expressed in terms of S_n as follows:

$$\bar{\epsilon} = \int_0^\infty dx \,\overline{\mathbf{j}\,\mathbf{E}} = - i \frac{3N}{2\pi p_F} \omega \int_0^{2\pi} d\phi \int d\eta\, \eta \int_\Delta^\infty d\epsilon \sum_{\sigma,\sigma'} \sigma' n_F\left(-\frac{\sigma'\epsilon}{T}\right)$$

$$\times \frac{u_0}{|1 + u_0 S_1 + u_0 S_{-1}|^2} \sum_{n=1}^{\infty} n \left\{ \left[\frac{\mathrm{Im}\, u_n}{|u_n|^2} \prod_{i=1}^{n} |S_i|^2 - \frac{\mathrm{Im}\, u_{-n}}{|u_{-n}|^2} \prod_{i=1}^{n} |S_{-i}|^2 \right] \right\}. \tag{4}$$

At small amplitudes $B \ll 1$, relation (3) makes it possible to calculate the absorption (4) in each order in the amplitude. This procedure has the properties of an improved perturbation theory: it is free of the divergences connected with the presence of a singularity in the state density near the energy gap.

The series that determines the absorption (4) contains, besides the thermal absorption, also a sum (which does not vanish at $T=0$) of contributions of multiphoton excitation-production processes:

$$\sum_{n=1}^{\infty} \Theta(\omega n - \Delta - \epsilon) \frac{n}{u_n} \prod_{i=1}^{n} |S_i|^2 . \tag{5}$$

The sum begins with the single-photon contribution, the n-th term is proportional to B^{2n-1}, and the corresponding enrichment in the general case is nonanalytic in the amplitude. The explicit expression that follows from (3)–(5) for the single-photon absorption takes near the threshold, in the principal approximation in B, the form

$$\bar{\epsilon} = \frac{3N}{\pi p_F}\,\omega \int do\, \eta \int_0^{\Omega} dE \,\frac{B^2\left(\frac{\Omega - E}{E}\right)^{1/2}}{\left(1 + 2B^2\sqrt{\frac{\Delta}{E}}\right)^2} \tag{6}$$

This expression corresponds to the results of the linear theory[1]:

$$\bar{\epsilon} = \frac{3\pi N}{32\, p_F}\,\omega\Omega \int_0^{\infty} \frac{dq}{q}\, p_S^2(q) \tag{7}$$

in the case when the denominator of the integrand in (6) is close to unity. The corresponding condition is obtained by substituting in B(3) the characteristic values of the glancing angle η, which are determined from the spatial-resonance condition[1] $\eta \sim \kappa(\Omega/\Delta)^{1/2}$; $p_S v_F/\Delta \ll (\Omega/\Delta)^{3/4}$. Near the threshold, the obtained criterion is quite stringent, so that the nonlinear situation (1) can be realized even at low amplitude. In the limit of strong nonlinearity, for a Pippard superconductor, we obtain in place of (7) the expression:

$$\bar{\epsilon} \sim \frac{N}{p_F}\,\omega\Omega\left(\frac{\Omega}{\Delta}\right)^{3/4} \kappa^{3/2}\, \frac{H}{H_C}\,. \tag{8}$$

The considered nonlinearity is substantially connected with the proximity to the threshold, where the single-quantum absorption, being the principal term in the expansion of the field, becomes small.

This makes it necessary to estimate the absorption due to the two-quantum processes. This estimate can also be based on Eqs. (3)–(5):

$$\bar{\epsilon} \sim \frac{N}{p_F}\,\omega\Delta\left(\frac{H}{H_C}\right)^4\left(1 + O\left(\sqrt{\frac{\Omega}{\Delta}}\,\ln\frac{H}{H_C}\right)\right) \tag{9}$$

and substitution of this estimate in (8) shows that the nonlinearity of the single-photon absorption sets in before the two-photon effects become significant.[2)]

Figures 1 and 2 show the amplitude and frequency characteristics of the surface

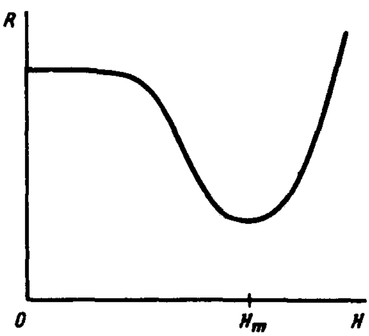

FIG. 1. Amplitude dependence of the surface resistance.

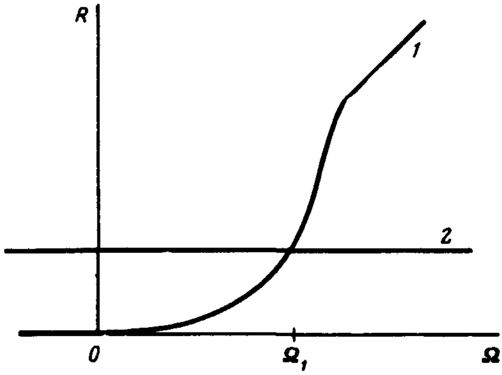

FIG. 2. Frequency dependence of the single-photon (1) and two-photon (2) absorption, $\Omega_1 \sim \left(\frac{H}{H_C}\kappa^{1/2}\right)^{1.7}\Delta$.

resistance of the superconductor, corresponding to formulas (7)–(9). As seen from these formulas, the nonlinearity of the single-photon absorption leads: 1) to the existence of a minimum on the amplitude dependence of the surface resistance at $H_m \sim H_C \kappa^{1/2}(\Omega/\Delta)^{0.6}$ (Fig. 1); 2) to a change of the frequency characteristic from a linear relation[1] to a $\Omega^{7/4}$ dependence in the region of (1) and to the absence of a kink at the threshold point (Fig. 2). It appears that a radiation power on the order of 10^{-1} W suffices for an experimental observation of the described effects. This power is readily attainable with modern submillimeter lasers.

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[1)]Since the phase-space volume occupied by the pair is small, their influence on other characteristics of the response of the superconductor is negligible.

[2)]The absorption connected with the oscillations, which is not taken into account in (3), is of the same order in H/H_C as the two-photon absorption.

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[2]V.P. Galaĭko and V.S. Shumeĭko, Fiz. Nizk. Temp. **1**, 1253 (1975) [Sov. J. Low Temp. Phys. **1**, 599 (1975)].