Energy feasibility of a relativistic Compton laser

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On the basis of the theory of the average motion of an electron in the field of two radio-frequency waves [A. V. Gaponov and M. A. Miller, Sov. Phys. JETP 7, 166 (1958); M. A. Miller, Izv. Vyssh. Uchebn Zaved. Radiofiz. 2, 438 (1959); Sov. Phys. JETP **9**, 1358 (1959)] the starting and working currents, as well as the efficiency of the generator are calculated. The permissible velocity scatter of the electrons and the degree of incoherence of the pumping is determined. It is shown that the energy parameters of the existing high-power radiation sources and of relativistic electron beams are sufficient, in principle, to realize a hydrodynamic regime of induced Compton scattering with an appreciable increase of the frequency at high efficiency.

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1. Although the problem of generation of coherent radiation by induced Compton scattering of an electromagnetic wave by relativistic electrons has been extensively discussed in the literature (see Ref. 2 and the review[3]), the problem of the optimal parameters of a Compton laser remains open. Yet, a nonlinear theory of such a generator can be constructed in elementary fashion by using the known results of Ref. 1. In fact, assume that the field of two waves $\mathrm{Re}\{\mathbf{E}_i \exp i(\omega_i t-\mathbf{k}_i\mathbf{r})+\mathbf{E}_s \exp i(\omega_s t-\mathbf{k}_s\mathbf{r})\}$, in an inertial reference frame K' that moves in the same direction as the translational motion of the electrons (the z axis on Fig. 1) with phase velocity $v_{\mathrm{ph}}=\Omega/\kappa_z$ of the combined wave, where

$$\Omega = \omega_s - \omega_i\,,\ \kappa = \mathbf{k}_s - \mathbf{k}_i\,,\ (\kappa_z = k_s\cos\phi_s + k_i\cos\phi_i),$$

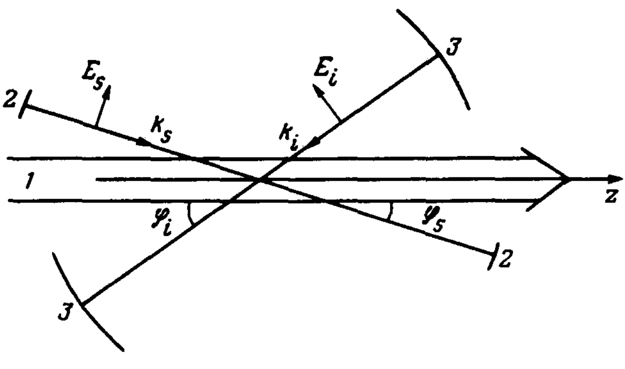

FIG. 1. Diagram of Compton laser: 1—electron beam; 2—mirrors of signal resonator; 3—mirrors of pump resonator.

has a single frequency: $\mathbf{E}'=\mathrm{Re}\{\mathbf{E}'(\mathbf{r}')\exp i\omega t'\}$. If the electron velocity v' in this system is much smaller than the speed of light c, then the averaged motion of the electron constitutes drift under the influence of the effective force $\mathbf{F}' = -(e^2/4m\omega'^2)\nabla'|\hat{\mathbf{E}}'|^2$, which at constant $\mathbf{E}_i$ and $\mathbf{E}_s$ is determined only by the combination wave[1] (the expression for $\mathbf{F}'$ is given here for the simplest case—when there is no longitudinal focusing magneto-static field). Accordingly, in the laboratory frame, the averaged motion of the electron is described by the same equations as in a traveling-wave tube (naturally, with the electric field of the traveling-wave tube replaced by the effective field of the combined wave):

$$\frac{du}{d\zeta} = -\sin\theta, \qquad \frac{d\theta}{d\zeta} = \Delta - u, \tag{1}$$

which in turn are identical to the equations of a pendulum $d^2\theta/d\zeta^2 - \sin\theta = 0$[1]. In (1) we have

$$\zeta = \kappa_z z \gamma_0^{-2}\sqrt{\mu a_i a_s}, \quad u = \sqrt{\mu/a_i a_s}, \quad w = (1 - \gamma/\gamma_0),$$
$$\theta = (\kappa \mathbf{r} - \Omega t), \ \Delta = \gamma_0^{(2)}(1 - v_{\rm ph}/v_0)/\sqrt{\mu a_i a_s}, \ a_{i,s} = e|\mathbf{E}_{i,s}|\sqrt{2}mc\omega_{i,s}$$
$$\mu = 1 + (\gamma_0 \phi_s)^2,$$

γ is the ratio of the electron energy to the rest energy mc^2, and v_0 is the initial velocity of the electron. For electrons making up a stationary monoenergetic beam, the boundary conditions for (1) take the form

$$u(0) = 0, \qquad \theta(0) = \theta_0, \qquad 0 \leqslant \theta_0 < 2\pi, \tag{2}$$

and the electronic efficiency is determined by the relations

$$\eta_e \approx \sqrt{(a_i a_s/\mu)}\,\bar{u}(\zeta_k), \quad \bar{u} = \frac{1}{2\pi}\int_0^{2\pi} u(\zeta_k, \theta_0)\, d\theta_0,$$

where

$$\zeta_k = \zeta(z = L) \equiv \pi M/\sqrt{\mu a_i a_s}$$

is a quantity proportional to the interaction length L.

2. According to (1), to form a compact bunch of electrons in the decelerating phase of the combined wave it is necessary that the kinematic ($\zeta_k\Delta$) and dynamic ($\zeta_k u$) displacements of the electrons relative to the wave be quantities of the order of π. The electronic efficiency is then maximal at field intensities that satisfy, in order of magnitude, the relations (compare with the results of numerical calculation shown in Fig. 2)

$$\sqrt{a_i a_s} \sim w \sim \eta_e \sim M^{-1}, \tag{3}$$

and the optimal phase velocity of the combined wave is determined by the expression $(v_0 - v_{\rm ph})/v_0 \sim M^{-1}\gamma_0^{-2}$. Accordingly, the optimal signal frequency deviates from exact

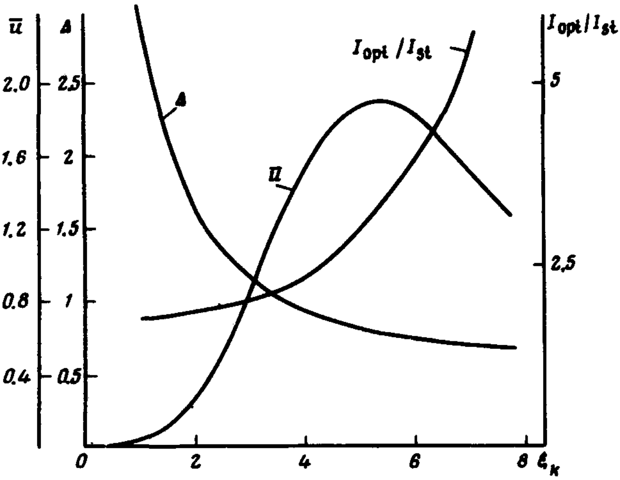

FIG. 2. Dependence of the optimal parameters and of the corresponding values of the relative electronic efficiency on the relative interaction length.

synchronism ($\Delta = 0$)

$$\omega_s^* = \omega_i \frac{1 + (v_0/c)\cos\phi_i}{1 - (v_0/c)\cos\phi_s} \tag{4}$$

by an amount on the order of $\omega_s^* M^{-1}$. It is clear from (3) that optimal intensities $\mathbf{E}_i$ and $\mathbf{E}_s$ of the pump and signal fields can be attained only in systems with sufficiently large ($M \gg 1$) interaction lengths (this condition is necessary also for the applicability of the averaging method).

In the stationary regime, the output power of the Compton laser greatly exceeds the consumed pump power (the number of scattered pump photons is equal to the number of signal photons, but the energy of the latter is approximately γ_0^2 times larger, so that $\omega_s \lesssim 4\gamma_0^2\omega_i$), so that the power balance equation takes the form

$$\frac{\omega_s}{Q_s} \frac{|\mathbf{E}_s|^2}{4\pi} S_s L_s \approx \eta_e m c^2 \gamma_0 (I/e), \tag{5}$$

where $Q_s = 2\pi(L_s/\lambda_s)\delta_s^{-1}$ is the quality factor, S_s is the area of the mirror, L_s is the length of the resonator, λ_s is the signal wavelength, δ_s is the loss coefficient of reflection from the mirror, and I is the beam current. In the small-signal approximation, when the electronic efficiency of the Compton laser is equal to

$$\eta_e = \sqrt{(a_i a_s/\mu)}\, \zeta_k^3\, \phi'(\theta_k), \quad \phi(\theta_k) = -(1 - \cos\theta_k)/2\theta_k^2, \theta_k\, \zeta_k\, \Delta, \tag{6}$$

we obtain from (5) an expression for the starting current

$$I_{st} = \frac{I_0}{\mu\gamma_0} \frac{\delta_s \delta_i}{M^3(P_i/P_0)} \frac{S_i S_s}{\lambda_i^2 \lambda_s^2} \frac{2}{\pi\phi'(\theta_k)}. \quad (7)$$

In (7), $I_0 = mc^3/e \approx 17$ kA, $P_0 = m^2c^5/e^2 \approx 8.7$ GW, P_i is the pump power, and S_i is the effective area on which the pump was focused.

Since the current corresponding to the maximum efficiency, $I_{opt} = I_{st}(\zeta_k^3/\bar{u})\phi'(\theta_k)$, is several times larger than the minimum starting current (Fig. 2), it follows that in the operating regime the generated-frequency band is close to the value $\Delta\omega_s \sim \omega_s^* M^{-1}$ determined by the fundamental tuning band ($2\pi \geqslant \theta_k \geqslant 0$). This band contains $\sim \gamma_0^2(L_s/L)$ longitudinal modes of the resonator.

3. The hydrodynamic formulas (6) and (7) remain in force so long as the synchronous-frequency scatter due to the initial scatter $\Delta\gamma$ of the electron energies, to the noncoherence of the pump $\Delta\omega_i$, and to the angle divergence of the incident and scattered waves $\Delta\phi_{i,s}$ (the energy and angle spreads of the photons) does not exceed the generated-frequency band: $\Delta\omega_s^* \lesssim \Delta\omega_s$. This limitation holds if the length parameter M does not exceed $(\gamma_0/\Delta\gamma)$ and

$$\Delta\omega_i \lesssim \omega_i M^{-1}, \quad \Delta\phi_i \lesssim 1/M \operatorname{tg}(\phi_i/2), \quad \Delta\phi_s \lesssim 1/M\gamma_0 f(\gamma_0\phi_s), \quad (8)$$

where $f(x) = 2x/(1+x^2)$. The more these conditions are violated, the larger the number of electrons exluded from synchronism with the combination wave and the efficiency decreases (the kinetic stage[2,3]).

The requirements imposed on the coherence of the pump are easy to satisfy even when powers from independent generators are added. As to the energy scatter of the electrons in high-density beams, which are needed to obtain the highest-frequency radiation to satisfy the corresponding restriction it is apparently necessary to use ion compensation of the space charge.

4. A relativistic Compton laser is most suitable for generation of radiation in a frequency band that is not readily accessible to classical devices of the ordinary type as well as for lasers. Thus, by using a beam with parameters close to those already realized, namely $\gamma_0 \sim (5\text{–}20)$, $\Delta\gamma/\gamma_S \sim (1\text{–}0.1)\%$, a current density $\sim(10^5\text{–}10^7)$ A/cm^2 and a CO_2-laser pump ($\lambda_i = 10.6\ \mu$m) with power $\sim(10^{10}\text{–}10^{11})$ W much lower than the attainable, it is possible to attain lasing with a tunable frequency in the range (2000–100) Å at the level $\sim 10^8$ W.

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[1)] The derivation of (1) is simplified if the invariance of F_z and the integral of the averaged energy,[1] equal in the l.s. to $\kappa E - \Omega_p =$ const, are taken into account. From the quantum point of view this integral reflects the fact that the changes of the electron energy and momentum in the elementary scattering act (with absorption of a pump quantum and emission of a signal quantum) are equal to $(\hbar\Omega)$ and $(-\hbar\kappa)$, respectively.

[1] A.V. Gaponov and M.A. Miller, Zh. Eksp. Teor. Fiz. **34**, 242 (1958) [Sov. Phys. JETP **7**, 168 (1958)]; M.A. Miller, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **2**, 438 (1959); Zh. Eksp. Teor. Fiz. **36**, 1909 (1959) [Sov. Phys. JETP **9**, 1358 (1959)].
[2] R.H. Pantell, G. Soncini, and H.E. Puthoff, IEEE J. Quantum Electron. **QE-4**, 905 (1968).
[3] V.L. Granatstein and P. Sprangle, IEEE Trans. Microwave Theory Tech. **MTT-25**, 245 (1977).