“Anomalous” temperature dependence of the rate of μ^+-meson polarization relaxation in some normal metals

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We obtain and solve equations that describe the “strange” behavior of the rate of relaxation of the polarization of μ^+ mesons in a number of metals. A new mechanism of this phenomenon is proposed. It is shown that in Nb and Bi the experiment cannot be attributed to trapping of muons by the impurities, but agrees fully with the hypothesis of trapping in different pores. Experiments are considered which make it possible to explain unambiguously the nature of the “strange” relaxation in an actual metal.

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As a rule in normal metals the rate of μ^+-meson polarization relaxation decreases monotonically with increasing temperature. Recently, however, a number of metals (Bi, Nb, Ta, Be) have revealed a nonmonotonic $\Lambda(T)$ dependence.[1-4] Usually this fact is interpreted as trapping of diffusing muons by the impurity centers. We present below the fundamental relations that explain this phenomenon, and describe a number of experiments that make it possible to explain the mechanism that leads to the “anomalous” behavior of $\Lambda(T)$.

It is known that muon polarization is observed in experiment. If the external magnetic field is perpendicular to the initial polarization, then $P_x(t) = \cos\omega t G(t) P(0)$ [the x axis is parallel to $\mathbf{P}(0)$]. Here $\omega = \gamma_\mu B$ is the muon precession frequency in the external field, and $G(t)$ is the relaxation function. All the published reductions of the experimental data for $G(t)$ were based on the well-known semi-empirical formula[5]

$$G(t) = \exp\left[-2\frac{\sigma^2}{\lambda^2}\left(e^{-\lambda t} + \lambda t - 1\right)\right], \tag{1}$$

in which in our case λ is the frequency of the muon jumps between interstices, and σ^2 is a parameter analogous to the second moment of the NMR line. For different metals σ^2 is of the order of 10^{10}–10^{12} sec^{-2} and is determined by the magnetic moments of the nuclei, by the structure of the crystal lattice, and by the point of localization of the muon.

In the derivation of (1) it is assumed that the change of the magnetic field at the muon upon diffusion is described by a Gaussian random process. Although actually the picture is more complicated, but when a muon diffuses over crystallographically equivalent pores, formula (1) yields at least a qualitatively correct result.

It is seen that the dependence of the polarization amplitude on the time is not exponential. It is possible, however, to introduce conditionally the concept of an effective crystallization rate Λ, defined as the reciprocal time during which the amplitude decreases by a factor e: $G(\Lambda^{-1})=e^{-1}$. Since λ increases with increasing temperature, it follows from (1) that Λ decreases monotonically with increasing T. This conclusion, however, as already indicated, contradicts the experimental results.[1-4] Thus, in niobium,[2,3] for example, at low temperatures Λ is constant, and then decreases and reaches a minimum at $T\sim 25$ K, after which it increases, stays constant for a while, and then decreases again.

This behavior of $\Lambda(T)$ can be explained by assuming that the crystal lattice contains at least two types of equilibrium positions for the muon. Birnbaum *et al.*[3] propose that the positions of the first type are interstices, and the second type is due to trapping of the muons by foreign impurities. The possibility of trapping by an impurity is evidenced, in particular, by the results of an experiment[6] in which they measured the rate of relaxation in pure aluminum and in the alloy Al + 1% Cu, and it was found that Λ in aluminum is small at all temperatures, whereas in the alloy the $\Lambda(T)$ dependence is qualitatively the same as in pure copper. One cannot, however, exlude completely the possibility that the positions of the types of Refs. 1 and 2 are crystallographically different interstices (for example, tetrapores and octapores). Indeed, in Ref. 7 they investigated by an NMR method hydrogen dissolved in niobium, and it was concluded that the hydrogen can be trapped in pores of either type.

To determine which of the indicated possibilities is actually realized, it is necessary to obtain formulas for $P(t)$. The polarization amplitude of the muons that diffuse only over positions of type (i) are determined by the function $G_i(t)$; as an approximation we can use for $G_i(t)$ an expression of the type (1). To take into account transitions between positions of different types, we introduce the following quantities: ν_{ik} is the probability of the transition from the position (k) into the position (i) per unit time $P_i(t)$ is the contribution made to the polarization by muons situated in positions (i) at the instant t. If W_i is the probability of muon trapping in the position (i) immediately after thermalization, then $P_i(0)=W_iP(0)$. It can be shown that the partial polarizations $P_i(t)$ satisfy the system of integral equations

$$P_i(t) = \exp(-\nu_i t)\, G_i(t) W_i\, P(0) + \sum_{k\neq i} \nu_{ik} \int_0^t \exp[-\nu_i (t-\tau)] G_i(t-\tau) P_k(\tau)\, d\tau, \tag{2}$$

where $\nu_i=\Sigma_{k=i}\nu_{ki}$ is the probability of the departure of the muon from the position of type (i) per unit time. A Laplace transformation reduces the system (2) to a system of linear algebraic equations. We present its solution for the case when there are positions of only two types:

$$P_{(p)} = P_1(p) + P_2(p)$$

$$= \frac{W_1 G_1(p+\nu_1) + W_2 G_2(p+\nu_2) + (W_1\nu_1 + W_2\nu_2)\, G_1(p+\nu_1)\, G_2(p+\nu_2)}{1-\nu_1\nu_2 G_1(p+\nu_1)\, G_2(p+\nu_2)}\, P(0) \tag{3}$$

Here

$$P(p) = \int_0^\infty P(t)\exp(-pt)\,dt,\quad G_i(p+\nu_i) = \int_0 G_i(t)\exp[-(p+\nu_i)t]\,dt.$$

In the analysis of (3) we shall for the sake of argument that the more stable positions are (2), i.e., $\nu_1 \gg \nu_2$. We consider the temperature region for which $\lambda_1 \gg \sigma_1$, and $\lambda_2 \approx \nu_2 \approx 0$, where λ_1 and λ_2 are the frequencies of the jumps over the positions (1) and (2), respectively. We have from (3) and (1)

$$P(t) \approx \left\{ W_1 \exp\left[-\left(\frac{2\sigma_1^2}{\lambda_1} + \nu_1\right)t\right] + W_2\exp(-\sigma_2^2 t^2) + W_1\nu_1\int_0^t \exp\left[-\sigma_2^2(t-\tau)^2 - \left(\frac{2\sigma_1^2}{\lambda_1} + \nu_1\right)\tau\right] d\tau \right\} P(0) \tag{4}$$

If the positions (2) are due to trapping by impurity atoms, then $W_2 \approx 0$ and $\nu_1/\lambda_1 \approx c$, where c is the relative concentration of the impurity. Then at $\nu_1 \ll \sigma_2$ the main contribution to formula (4) is made by the first term, and we obtain for the relaxation rate

$$\Lambda \approx \frac{2\sigma_1^2}{\lambda_1} + \nu_1 = \frac{2\sigma_1^2}{\lambda_1} + c\,\lambda_1 \tag{5}$$

It follows therefore that at $\lambda \approx \sigma_1\sqrt{2/c}$ the relaxation rate is minimal, with $\Lambda_{\min} \approx 2\sigma_1\sqrt{2c}$. Thus we have the following: 1) at the minimum point λ is proportional to $c^{-1/2}$, i.e., when the impurity concentration is decreased, the position of the minimum of Λ should shift towards higher temperatures; 2) $\Lambda_{\min} \ll \sigma_1$. These conclusions contradict the results obtained for niobium and bismuth.[1-3] We used in the experiments samples with impurity concentrations of approximately 10^{-4}, and it was found that $\Lambda_{\min}/\sigma_1 \approx 0.35$–$0.6$, whereas the calculated value is ~ 0.03. Therefore positions (1) and (2) in this case are apparently pores of different types. For a conclusive answer to the question it is necessary to perform more detailed experimental investigations, the most obvious of which is the measurement of the dependence of Λ on the impurity concentration.

At higher temperatures, the condition $\nu_1 \gg \sigma_2$ is satisfied and it follows from (3) that $P(t) \approx \exp(-\sigma_2^2 t^2)P(0)$. Measurements of the parameter σ^2 makes it possible to determine where the muons are localized in this case. Finally, with further increase of T, diffusion over the pores (2) is turned on (or else the muons begin to break away from the impurity centers), and Λ decreases. Experimental data on beryllium[4] in which the $\Lambda(T)$ dependence exhibits two plateaus separated by a minimum, can be explained if it is assumed that three types of equilibrium positions exist.

It seems to us that the formulas presented above uncover new possibilities for the investigation of both the behavior of the muons in metals and of the potential relief of the crystal lattices.

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