Electric domains in metals at low temperatures

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It is shown that spontaneous onset of electric-field domains, due to the N-shaped current-voltage characteristic (CVC), can occur in metals at low temperatures. Several mechanisms that produce N-shaped CVC are indicated. The main features of electric domain structures in metals are considered.

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1. The Gunn effect in semiconductors with N-shaped current-voltage characteristic (N-CVC) is by now well known. It is due to instability of the homogeneous distributions of the electric field $\mathscr{E}$ and of the charge on the CVC section with $j' \equiv \partial j/\partial \mathscr{E} < 0$ (j is the current density), and constitutes a spontaneous onset of electric domains that move along **j**.[1] In the present paper we show that analogous phenomena are possible in metals at low temperatures, and discuss their main features.

2. a) The simplest ("temperature") mechanism of formation of N-CVC in metals is realized when the metal is Joule-heated to a characteristic temperature T at which the contribution of the electron-phonon scattering to the electric conductivity $\sigma(j = \sigma[T(\mathscr{E})\mathscr{E}]$ remains $\sim \sigma_0 = \sigma|_{T=0}$. In the typical situation $d\sigma/dT < 0$, the heat-balance condition $q(T) = \sigma(T)\mathscr{E}^2 d$ (q is the heat flux from the metal at a given temperature and at a specified heat-transfer conditions, d is the sample thickness) yields for the onset of N-CVC the criterion $d[\sigma(T)q(T)]/dT < 0$. This inequality is easily realized.

b) New possibilities for the formation of N-CVC appear under conditions of the so called coherent magnetic breakdown (CMB), when the dissipative processes do not disturb the magnetic-breakdown energy spectrum, i.e., the inequality $\omega_H \tau^* > 1$, is satisfied, where ω_H is the characteristic Larmor frequency and τ^* is the lifetime of the electron in stationary states of the magnetic breakdown (MB) spectrum. (Owing to the interaction of the electron with the dislocation strain fields and with the phonons, τ^* can generally speaking be less than the electron-impurity relaxation time τ_{imp}.[2])

We shall deal with a frequently encountered case, when all the classical electron orbits in the magnetic field $\mathbf{H} = \{0,0,H\}$ are closed and the MB forms from them in **p**-space an "MB layer" (of width δp) of periodic MB configurations which are open along one of the reciprocal-lattice vectors $\mathbf{b} = \{0,b,0\}$; (the current $\mathbf{j} = \{j,0,0\}$). The energy spectrum inside the MB layer consists of the magnetic bands

$E_n(p_z,p_x) = E_n(p_z,P_x + \Delta P_x)$—broadened Landau levels, where n is the number of the magnetic band, p_z is the projection of the momentum on the **H** direction, and the period is $\Delta P_x = 2\pi e\hbar H/cb \sim (10^{-4} - 10^{-5})b$. At $w(1-w) \sim 1$ [$w = w(H)$ is the MB probability] the characteristic width of the magnetic bands and the distance between them is $\sim \hbar\omega_H$ (Fig. 1). In the stationary state the average transverse electron velocity

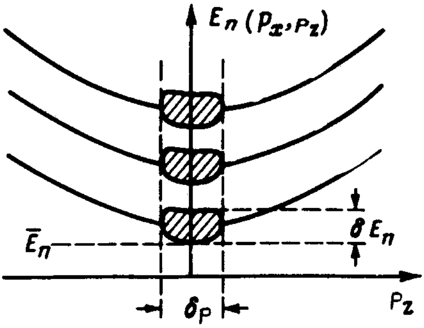

FIG. 1.

from the MB layer is $\langle v_x \rangle = \partial E_n/\partial P_x$ and is proportional to the Fermi velocity v_F while $\langle v_y \rangle = 0$. Outside the MB layer we have $\langle v_{x,y} \rangle = 0$, and consequently the MB layer makes the main contribution to j at $\beta \equiv (b/\delta p)(1/\omega_H^2\tau^*\tau_{\text{imp}}) \ll 1$.

One of the CMB mechanisms of the nonlinearity of $j(\mathscr{E})$ is based on the localization of the electron in the field $\mathscr{E} = \{\mathscr{E},0,0\}$ inside quantum "traps"[3] of width $\Delta x \sim \hbar\omega_H/e\mathscr{E}$; this localization is due to the energy-conservation law $e\mathscr{E}x + E_n(p_z,P_x) = \text{const}$. If the parameter of this "dynamic" nonlinearity $\gamma_{\text{dyn}} = \Delta x/l^* \ll 1 (l^* = v_F\tau^*)$, then at $\beta \ll 1$ we have $j \sim \gamma_{\text{dyn}}^2 \mathscr{E} \sim 1/\mathscr{E}$. We see therefore that the N-CVC arises if $\gamma_{\text{dyn}} \lesssim 1$. It turns out that the described "dynamic" mechanism can be realized in experiment if the angle between H and the plane perpendicular to b is of the order of 10^{-3}–10^{-2} rad.

If $\gamma_{\text{dyn}} \gg 1$, then the cause of the nonlinearity of $j(\mathscr{E})$ can be diffusion,[3] due to the field $\mathscr{E}$, of the electrons over the energies $E_n(p_z,P_x)$. We now formulate the main idea of the "diffusion" mechanism, assuming that $E_{n+1}^{(\min)}(H) > E_n^{(\max)}(H)$, where $E_n^{(\max)}$ and $E_n^{(\min)}$ is the absolute maximum and minimum of $E_n(p_z,P_x)$ within the MB layer. This situation can arise in metals with small $\delta p \lesssim \sqrt{e\hbar H/c}$ when H is parallel to the symmetry axis (examples: Be, Ru, Al). In this case the main contribution to j (at $\alpha \ll 1$) is made by electrons with energies from the intervals $\Delta_n = [E_n^{(\min)}, E_n^{(\max)}]$, where $\langle v_x \rangle \neq 0$ Since the diffusion flux through the boundary Δ_n is small (in terms of the parameter β), it follows that $\tau_{\text{dif}} \sim (b_0/\delta p)\tau^*\delta E_n^2/(e\mathscr{E}l^*)^2 \ll \tau_E$, which is the characteristic time of the electron-phonon energy relaxation (τ_{dif} is the characteristic time of the diffusion change of E_n by an amount $\delta E_n = E_n^{(\max)} - E_n^{(\min)} \sim \hbar\omega_H$) the diffusion "intermixes" practically uniformly the electrons within Δ_n, and the derivative df/dE on the intervals Δ_n turns out to be proportional to $\tau_{\text{dif}}/\tau_E \sim 1/\mathscr{E}^2$ [$f(E)$ is the electron energy distribution function]. The current j connected with df/dE by the usual relation of the linear theory, is in this case $\sim 1/\mathscr{E}$. Analysis shows that the "diffusion" mechanism leads to N-CVC in the entire region $\tau_{\text{dif}}/\tau_E \leqslant \gamma_{\text{cr}} \sim 1$, $\beta \leqslant \beta_{\text{cr}} \sim 1$.

3. In metals, in contrast to semiconductors, the spatially inhomogeneous distributions of $\mathscr{E}$ which arises at $j_0(\mathscr{E}_1) < j < j_0(\mathscr{E}_2)$, [$\mathscr{E}_{1,2}$ are the N-CVC extremal points $j_0(\mathscr{E})$, $\mathscr{E}_1 < \mathscr{E}_2$] develop under conditions of electroneutrality when the current density $j = j(t)$ (t is the time) is constant along the entire sample. In the case of weak spatial and temporal dispersion ($\lambda = (\mathscr{E}_2 - \mathscr{E}_1)/\mathscr{E}_1 \ll 1$) the equation for $\mathscr{E}(x,t)$ (x is the coordinate along $\mathbf{j}$) takes the form

$$-\sigma_1 t_0 \frac{\partial \mathscr{E}}{\partial t} + \sigma_1 l_0^2 \frac{\partial^2 \mathscr{E}}{\partial \xi^2} - j_0(\mathscr{E}) = j(t), \qquad \xi = x - st, \quad \sigma_1 = j_1/\mathscr{E}_1 . \tag{1}$$

The constants $t_0 > 0$ and l_0 are, respectively, the characteristic temporal and spatial scales of the nonlocal functional that relates j with $\mathscr{E}$; the constant s has the meaning of the velocity of the electric domain.

We present now estimates for l_0, t_0, and s. 1) "Temperature" mechanism: $t_0 \sim dTC_v/q$, $l_0 \sim \sqrt{\kappa Td/q}$, $s \sim \alpha \mathscr{E}_1/C_v$; C_v is the heat capacity of the metal per unit volume, κ is the thermal conductivity of the metal along j, $\alpha(T)$ is the "thermoelectric" coefficient of proportionality between j and $\partial T/\partial x$. At $l_0 \lesssim L_0 \sim 1$ cm (L_0 is the length of the sample along j), $q \lesssim 1$ W/cm^2, $\tau_{\text{imp}} \sim 10^{-8} - 10^{-9}$ sec, and at the "usual" values $\alpha(T) \sim (k^2T/e\epsilon_F)$ (ϵ_F is the Fermi energy) the value of s turns out to be $\lesssim 10^2$ cm/sec. 2) "Dynamic mechanism: $t_0 \sim \tau^*, l_0 \sim v_F\tau^*, s \sim v_F \sim 10^8$ cm/sec. 3) "Diffusion" mechanism:

$$t_0 \sim \tau_E,\ l_0 \sim v_F\sqrt{\tau_{\text{imp}}\tau_E(\delta p/b)},\ s \sim v_F\sqrt{(\tau_{\text{imp}}/\tau_E)}\delta p/b \sim 10^6 \text{ cm/sec}.$$

The characteristic values of $\mathscr{E}_{1,2}$ for the temperature mechanism are 10^{-6}–10^{-7} cgs esu; for the CMB mechanisms at $q \ll 1$ W/cm^2 and $d \lesssim 10^{-1}$ cm we have $\mathscr{E}_{1,2} \sim 10^{-4}$–$10^{-5}$ cgs esu.

4. Equation (1) admits of a stable self-similar solution of the soliton type $\mathscr{E}(x,t) \equiv \mathscr{E}_{\text{sol}}(\xi)$, $\mathscr{E}_{\text{sol}}(\xi) \to 0$ as $\xi \to \pm\infty$, if the metallic sample with N-CVC (of length $L_0 \gg \Delta\xi \sim l_0\lambda^{-1}$ is the characteristic dimension of the soliton) is connected in a circuit with a constant voltage source and is shunted by a conductor of sufficiently low resistance. These solitons are analogous to Gunn domains in semiconductors.

In metals where the values of $\mathscr{E}_{1,2}$ are smaller by 6 to 8 orders of magnitude than in semiconductors, a new contactless organization of the experiment is possible: in a sample with N-CVC, in the form of a cylindrical tube with arbitrary cross section and with contour length $L_0 \gg d$, the field $\mathscr{E}$ is induced by an alternating magnetic-field flux $\phi(t)$. The latter can be produced in any manner within the framework of the adiabaticity condition: $\dot\phi(\phi \ll s/L_0)$, $t_0^{-1}(\dot\phi = d\phi/dt)$. In this case $\mathscr{E}(\xi,t)$ satisfies the equation (1) (x is the arc length along the contour of the cylinder cross section) and the boundary conditions

$$\mathscr{E}(\xi, t) = \mathscr{E}(\xi + L_0, t),\ L_0^{-1}\int_0^{L_0} d\xi'\mathscr{E}(\xi' t) = \bar{\mathscr{E}}(t) \equiv -\dot\phi/L_0 c.$$

In the "contactless" situation, in contrast to the traditional (semiconductor) organization of the experiment, it becomes possible to investigate the kinetics of the development of electric domain structures. Analysis shows that in this case there exist three regions of values of the parameters $\tilde{\mathscr{E}}$ and $z=(2\pi l_0)^2/L_0^2$ with qualitatively different structure of the field $\mathscr{E}(\xi,t)$. In the region I (Fig. 2) a homogeneous distribution of $\mathscr{E}$

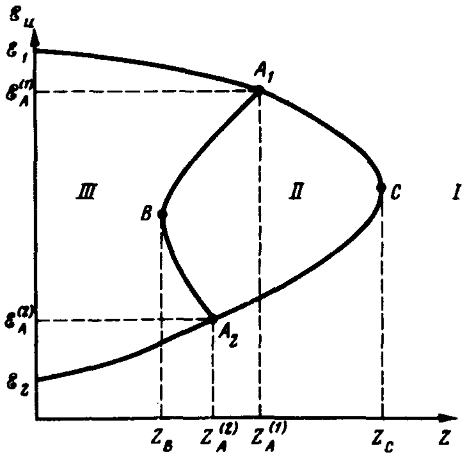

FIG. 2.

is stable. In the "domain" region II, any initial distribution of $\mathscr{E}$ develops into a *periodic* self-similar solution $\mathscr{E}(x,t) \equiv \tilde{\mathscr{E}}(\xi) = \tilde{\mathscr{E}}(\xi + L_0)$, which is uniquely defined together with the current j by specifying $\tilde{\mathscr{E}}(t)$. In the "turbulent" region III, the field $\mathscr{E}(x,t)$ is substantially irregular with respect to x and t. The boundary of the region I at $\lambda \ll 1$ is determined by the equation

$$\sigma_1 z = j'(\mathscr{E}); \; z_B \sim z_C = \mathrm{Max}\,|j'|/\sigma_1; \; \sigma_1(z_C - z_A^{(1,2)}) = 3/40\,(d^3 j/d\mathscr{E}^3)(\mathscr{E}_2 - \mathscr{E}_1)^2; \; \mathscr{E}_A^{(1)} - \mathscr{E}_A^{(2)} = \sqrt{3/5}\,(\mathscr{E}_2 - \mathscr{E}_1).$$

The region II of the stable solutions $\tilde{\mathscr{E}}(\xi)$ that are periodic in t (with a period $2\pi L_0/s$ exists also at $\lambda \sim 1$ and $d \sim L_0$. In this case

$$Z_A^{(1,2)} \; Z_B, \; Z_C \sim 1, \; (\mathscr{E}_A^{(1)} - \mathscr{E}_A^{(2)})/\mathscr{E}_1 \sim \lambda.$$

Periodic oscillations of $\mathscr{E}$ always have a characteristic amplitude $\sim \mathscr{E}_2 - \mathscr{E}_1$.

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